53 (MA 101) ENMA

2018

ENGINEERING MATHEMATICS–I

Paper : MA 101 (Back)

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* Find degree and order of the following differential equations : 2×2=4

(i) $$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$$

(ii) $$\frac{d^2y}{dx^2} - \sin\left(\frac{dy}{dx}\right) + y = 0$$

(b) Test the following series : 6

$$\sum_{n=1}^{\infty}\left(\frac{n^{5/2}}{n^4+3n^3}\right)$$

(c) Find the area bounded by the curve $xy^2 = 4a^2(2a - x)$ and its asymptote. 5

(d) Find the perimeter of the circle $x^2 + y^2 = a^2$. 5

2. (a) Form the differential equations : 4+4=8

(i) $y = ae^{3x} + be^{x}$

(ii) $y = e^{x}(A\cos x + b\sin x)$

(b) Test the absolute convergency of the series 6

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n\left(n^{\frac{1}{3}} + 1\right)}.$$

(c) Evaluate : 3+3=6

(i) $\int_0^{\pi/6} \cos^4(3\theta)\sin^3(6\theta)\,d\theta$

(ii) $\int_0^{\pi} \sin^4\left(\frac{\theta}{2}\right)\cos^3\left(\frac{\theta}{2}\right)d\theta$.

3. *(a)* Solve the following differential equations : 3×4=12

(i) $\frac{dy}{dx} = \frac{3xy + y^2}{3x^2}$

(ii) $(x + y - 10)dx + (x - y - 2)dy = 0$

(iii) $3\frac{dy}{dx} + 3\frac{y}{x} = 2x^4y^4$.

(b) State Cauchy's root test. Using it discuss the convergency of the following : 2+6=8

$$\frac{2}{1^2}x + \frac{3^2}{2^3}x^2 + \frac{4^3}{3^4}x^3 + \ldots\ldots \text{to } \infty$$

4. *(a)* Solve : 5

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = \sin h(x)$$

(b) Find the maximum and minimum of the function,

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20.$$ 6

(c) If $y = e^{\tan^{-1}(x)}$, show that

$$(1 + x^2)y_{n+2} + [2(n+1)x - 1]y_{n+1} + n(n+1)y_n = 0.$$

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(d) Find the equation of the sphere passing through three points $(0,0,0), (1,-1,0), (2,0,-2)$ and $(0,1,-2)$. 5

5. (a) Solve the following simultaneous equation : 5

$$\frac{dx}{dt} + 2x - 3y = 0$$

$$\frac{dy}{dt} - 3x + 2y = 0.$$

(b) Find the equation of the plane through the line $\frac{x-3}{5} = \frac{y+2}{-3} = \frac{z}{2}$ parallel to the line, $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z+4}{3}$. 6

(c) Expand $log(\sin x)$ in power of $(x-3)$. 3

(d) Find all the asymtotes of the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$. 6

6. (a) Show the equation of the plane through the points (α, β, γ) parallel to the plane $ax + by + cz + d = 0$ is $a(x-\alpha) + b(y-\beta) + c(z-\gamma) = 0$. 6

(b) If 1,2,3 are the direction ratios of a line through the origin, find the co-ordinates of a point on the line at a distance 5 from the origin. 4

(c) Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$. 5

(d) Use Taylor's series expansion to compute the value of *cos* 32°, correct to four decimal places. 5