2014

ENGINEERING MATHEMATICS – I

Paper : MA 101

Full Marks : 100

Pass Marks : 30

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Using limit comparison test, show that the series $\frac{2^p}{1^q}+\frac{3^p}{2^q}+\frac{4^p}{3^q}+\cdots$ is convergent if $p-q+1<0$ and divergent if $p-q+1\geq 0$. 5

   (b) If $y=a\cos(\log x)+b\sin(\log x)$, show that

   (i) $x^2y_2+xy_1+y=0$

   (ii) $x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$

   3+3=6

(c) Find the area of the surface formed by the revolution of the parabola $y^2 = 4ax$ about the x-axis by the arc from the vertex to one end of the latus rectum. 7

(d) Find the equation of the plane passing through the point (1, 2, 3) and parallel to the plane $3x+4y-5z=0$. 2

2. (a) If $I_n = \int sin^n x\, dx$, then show that

$$I_n = \frac{n-1}{n} I_{n-2} - \frac{1}{n} sin^{n-1} x.cos\, x$$ 6

(b) Prove that for the curve $x^{2/3} + y^{2/3} = a^{2/3}$, the portion of the tangent intercepted between the axes is of constant length. 7

(c) Find the order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3}$.

Find the differential equation for the family of curves $x^2+y^2-2ay=0$, where a is an arbitrary constant. 2+5=7

3. *(a)* Show that an absolutely convergent series is always convergent. Is the converse true ? Justify. 3+2=5

(b) Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ of the curve $x^3 + y^3 = 3axy$. 5

(c) Expand $e^{\sin x}$ by Maclaurin's theorem. 5

(d) Find the equation of the plane which cuts the axes at the points A, B, C, if the centroid of the triangle ABC is (p, q, r). 5

4. Solve the following differential equations : 5×4=20

(i) $\frac{dy}{dx} = \frac{y - x}{y + x}$

(ii) $2xy.\frac{dy}{dx} = x^2 + y^2$

(iii) $\frac{dy}{dx} + \frac{y}{x} = y^2$

(iv) $x\frac{dy}{dx} - y - 2x^3 = 0$

5. *(a)* State Leibnitz test. Show that the series

$$\frac{1}{1+a^2}-\frac{1}{2+a^2}+\frac{1}{3+a^2}-\frac{1}{4+a^2}+\cdots$$

in convergent. 2+4=6

(b) State Euler's theorem on homogeneous function. Using this show that if

$$u=sin^{-1}\left(\frac{x^2+y^2}{x+y}\right),\text{ then}$$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=tan\,u.$$ 2+5=7

(c) Find the equation of the plane which passes through the point (2, 1, 4) and is perpendicular to the planes

$9x-7y+6z+48=0$ and $x+y-z=0$ 7

6. *(a)* Solve : 5×3=15

(i) $\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=x$

(ii) $\dfrac{d^2y}{dt^2} - a^2y = e^{at}$

(iii) $\dfrac{d^2y}{dx^2} + a^2y = \sin ax$

(b) Find all the asymptotes of the curve

$x^2y^3 + x^3y^2 = x^3 + y^3$. 5

7. (a) Is the series $\sum_n \dfrac{1.2.3.\cdots.n}{7.10.\cdots(3n+4)}$ convergent ? Justify your answer. 5

(b) Define direction cosines of a line. Find the direction cosines of a line equally inclined to $\overrightarrow{OX}$, $\overrightarrow{OY}$, $\overrightarrow{OZ}$. 2+3=5

(c) Solve $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 2y = 0$ subject to the condition $x = 0$, $y = 3$ and $\dfrac{dy}{dx} = 0$. 5

(*d*) Examine the following series stating the test of convergence applied

$$\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-1}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-1}+\cdots$$

5
