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53 (MA 201) ENMA-II

2014

ENGINEERING MATHEMATICS II

Paper : MA 201

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. (a) If $\frac{d\vec{u}}{dt}=\vec{w}\times\vec{u}, \frac{d\vec{v}}{dt}=\vec{w}\times\vec{v}$, show that

$$\frac{d}{dt}(\vec{u}\times\vec{v})=\vec{w}\times(\vec{u}\times\vec{v}).$$ 6

(b) If $f(x)=\begin{cases} x+x^2 & \text{for} \quad -\pi<x<\pi \\ \pi^2 & \text{for} \quad x=\pm\pi \end{cases}$,

Show that

$$x+x^2=\frac{\pi^2}{3}+\sum_{n=1}^{\infty}(-1)^n\left\{\frac{4}{n^2}cos\,nx-\frac{2}{n}sin\,nx\right\}$$ 6

Contd.

(c) (i) Show that positive odd integral powers of a skew-symmetric matrix are skew symmetric while positive even integral powers are symmetric. 4

(ii) If A is any real skew-symmetric matrix such that $A^2 + I = 0$, show that A is of even order. 4

2. (a) Find the median from the following data :

Class Internal	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45
Frequency	29	195	241	117	52	10	6	3	2

6

(b) If $\vec{r} = ae^{nt} - be^{-nt}$, where a and b are constant vectors then prove that $\dfrac{d^2\vec{r}}{dt^2} = n^2\vec{r}$. 6

(c) Find the Fourier expansion for $f(x)$ if

$$f(x) = \begin{cases} -\pi & ; \quad -\pi < x < 0 \\ x & ; \quad 0 < x < \pi \end{cases}$$

and deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \text{.......} \text{ to } \infty = \frac{\pi^2}{8}.$$ 8

3. *(a)* If the product of two non-zero square matrices is a zero matrix, show that both of them must be singular matrices. 5

(b) A pair of dice is rolled twice. Let X denote the number of times "a total of 9" is obtained. Find the mean and variance of the random variable X. 7

(c) *(i)* If $\vec{A} = x^2yz\,\hat{i} - 2xz^3\,\hat{j} + xz^2\hat{k}$,

$$\vec{B} = 2z\hat{i} + y\hat{j} - x^2\hat{k},$$

find the value of $\frac{\partial^2}{\partial x \partial y}\left(\vec{A} \times \vec{B}\right)$ at $(1, 0, -2)$. 4

(ii) If $f(x, y, z) = 3x^2y - y^3z^2$, find g rad f at the point $(1, -2, -1)$. 4

4. *(a)* Show that the matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ satisfies Cauchy-Hamilton theorem, Hence the value of A^{-1} and A^2. 6

(b) (i) If $u = x^2 - y^2, v = 2xy$ and $x = r\cos\theta$, $y = r\sin\theta$, find $\frac{\partial(u,v)}{\partial(r,\theta)}$. 3

(ii) Prove that $JJ' = 1$, where

$$J = \frac{\partial(u,v)}{\partial(x,y)},\ J' = \frac{\partial(x,y)}{\partial(u,v)}.$$ 3

(c) Let X have the probability density function (p.d.f.)

$$f(x) = \begin{cases} \frac{1}{2}e^{-x/2} & , \quad x > 0 \\ 0 & , \quad \text{otherwise} \end{cases}$$

find its moment generating function (m.g.f.) and hence find its mean and variance. 8

5. (a) If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1}x + \tan^{-1}y$. Show that they are functionally related. So, find its relationship. 7

(b) Reduce the matrix A to its normal form where

$$A = \begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

and find the rank and nullity of A. 6

(c) Evaluate $\int_C \vec{F}.d\vec{r}$, where $\vec{F} = xy\hat{i} + \left(x^2 + y^2\right)\hat{j}$ and curve C is the arc of $y = x^2 - 4$ from (2, 0) to (4, 12). 7

6. (a) Compute the inverse of

$$X = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -2 & 1 & -1 & -2 \\ -4 & -2 & -3 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$ by elementary transformation method. 6

(b) Find the half-cosine series for the function $f(x) = (x-1)^2$ in the interval $0 < x < 1$. Hence show that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... \text{ to } \infty = \frac{\pi^2}{8}$$ 8

(c) Let A and B be the two events such that $P(A)=3/4$ and $P(B)=5/8$. Show that

(i) $P(A\cup B)\geq 3/4$

(ii) $3/8\leq P(A\cap B)\leq 5/8$

3+3=6

7. (a) (i) Prove that $\beta(m, 1/2)=2^{2m-1}\beta(m, m)$

(ii) Prove that $\Gamma(1/2)=\sqrt{\pi}$. 4+5=9

(b) Obtain the Fourier series for $f(x)=\pi x$ in $0\leq x\leq 2$. 4

(c) Evaluate $\iint_S \{(x+z)\,dy\,dz+(y+z)\,dz\,dx+(x+y)\,dx\,dy\}$ where S is the surface of the sphere $x^2+y^2+z^2=4$. 4

(d) A five figure number is formed by the digits 0, 1, 2, 3, 4 (without repetition). Find the probability that the number formed is divisible by 4. 3

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