Total number of printed pages–4

53 (IT 717) GRTH

2021

(Held in 2022)

GRAPH THEORY

Paper : IT 717

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Define bipartite graph and complete bipartite graph. 4

   (b) Describe independent set and edge cover of a graph. 6

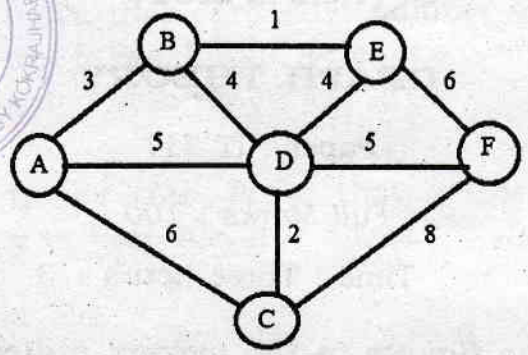
   (c) Define walk, path and circuit in a graph. 4

   (d) Show that the maximum number of edges in a simple graph with n vertices is $n(n-1)/2$. 6

Contd.

2. *(a)* Show the steps of Kruskal's and Prim's algorithm to find a minimum spanning tree of the graph shown below :

20

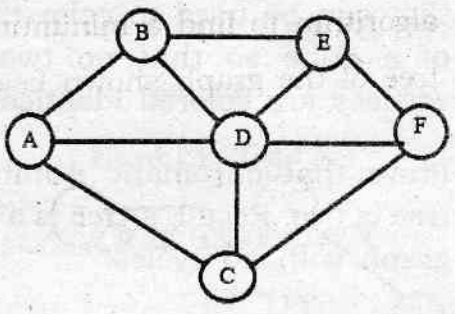


3. *(a)* What is a Hamiltonian graph ? Prove that in a simple graph G with $n(\geq 3)$ vertices and if the degree of each vertex $d(v) \geq n/2$ then G is a Hamiltonian.

12

(b) Show that a graph is bipartite if it does not have any odd cycle. 8

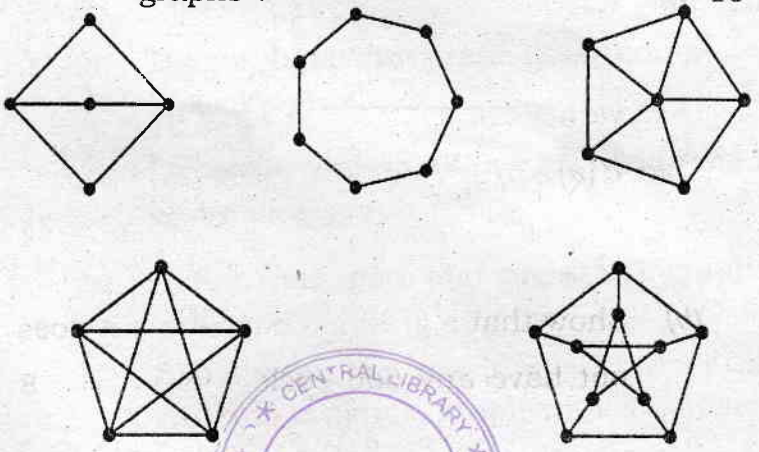
4. Consider the graph G in the figure below :



Find the BFS and DFS tree. 10+10=20

5. You are given a 4 liter and a 3 liter jug. Measure 2 liter of water using BFS and DFS. 10+10=20

6. *(a)* Find a chromatic number of the following graphs : 10



(b) What is the smallest number of colors that can be used to color the vertices of a cube so that no two adjacent vertices are colored identically? 7

(c) Prove that chromatic number of any tree is two. Recall, a tree is a connected graph with no cycles. 3

7. (a) Define tree. Prove that a tree with n vertices has $n-1$ edges. 10

(b) Describe the Chinese postman problem. 10