53 (IE 811) OPRE

2014

OPERATION RESEARCH

Paper : IE 811

Full Marks : 100

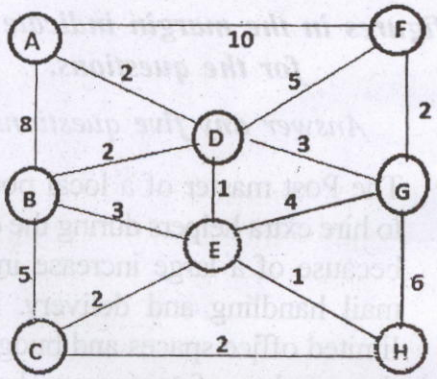
Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* The Post master of a local post office wishes to hire extra helpers during the depawali season because of a large increase in the volume of mail handling and delivery. Because of the limited office spaces and budgetary condition, the number of temporary helpers must not exceed 10. According to the past experience a man can handle 300 letters and 80 packages per day, on the average and a woman can handle 400 letters and 50 packages per day. The Post master believes that the daily volume of extra mail and packages will be not less than 3400 and 680 respectively. A man receives Rs 25/- a day and a woman receives Rs 22/- a day. How many men and women helpers should be hired to keep the pay roll at a minimum ? 10

(b) The national park service plans to develop area for the tourism. Eight locations in the area are designated for automobile access. These sides and distances (in miles) between them are shown in the network diagram. Determine the most economical miles of roadway required to provide the desired accessibility by the park service. 5



(c) Solve the following assignment problem to find the maximum total expected sale :

Area		*I*	*II*	*III*	*IV*
	A	42	35	28	21
Salesman	*B*	30	25	20	15
	C	30	25	20	15
	D	24	20	16	12

5

2. *(a)* Solve the following linear programming problem :

Maximize $Z = 2x_1 + 3x_2$

Subject to $-x_1 + 2x_2 \leq 4$

$x_1 + x_2 \leq 6$

$x_1 + 3x_2 \leq 9$

and x_1, x_2 are unrestricted. 14

(b) What are the differences between transportation and an assignment problem ? Find an initial feasible solution of the following transportation problem by North-West corner rule. 6

Market	Plant				Required at Market
	P_1	P_2	P_3	P_4	
M_1	19	14	23	11	11
M_2	15	16	12	21	13
M_3	30	25	16	39	19
Available at plant	6	10	12	15	43

3. *(a)* Find the dual of the following L.P.P.

Minimize $Z = x_1 + x_2 + x_3$

Subject to $x_1 - 3x_2 + 4x_3 = 5$

$x_1 - 2x_2 \leq 3$

$2x_2 - x_3 \geq 4$ 3

and $x_1, x_2 \geq 0$; x_3 is unrestricted in sign.

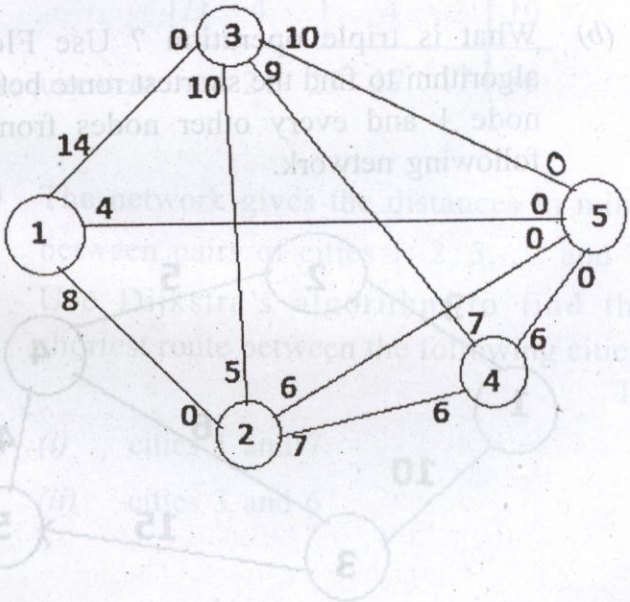
(b) There are five jobs *A*, *B*, *C*, *D* and *E* and these are performed on five machine centres *I*, *II*, *III*, *IV* and *V*. One job is to be allocated to machine centre, though each machine is capable of doing any job at different cost given by the matrix below :

	I	*II*	*III*	*IV*	*V*
A	25	29	31	42	37
B	22	19	35	18	26
C	39	38	26	20	33
D	34	27	28	40	32
E	24	42	36	23	45

(i) Find an allocation of jobs to the machine centre so that the total cost of processing is a minimum.

(ii) If the machine V goes out of order, which job will then be allocated to the rest of the centre at a minimum cost. 4+4=8

(c) Use maximal flow algorithm to obtain the total amount of flow and the amount of flow between the nodes (1,2), (2,3) and (2,5) in the following network : 9



4. *(a)* What do you mean by linearly independent set and linearly dependent set of vectors ? Using this concept define basic feasible solution. Find all the basic feasible solutions of the following linear Programming Problems.

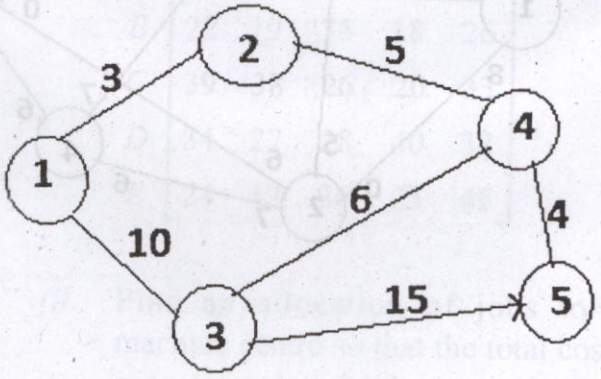
$$\text{Max} \quad Z = 2x_1 + 3x_2 + 4x_3 + 7x_4$$

$$\text{s.t} \quad 2x_1 + 3x_2 - x_3 + 4x_4 = 8$$

$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$

$$\text{and } x_i \geq 0 \ \forall \ i = 1,2,3,4$$ 10

(b) What is triple operation ? Use Floyd's algorithm to find the shortest route between node 1 and every other nodes from the following network. 10



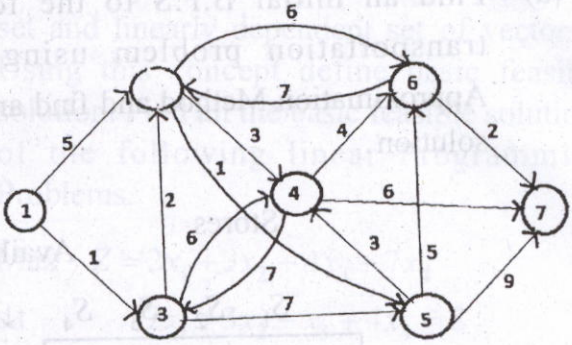
5. *(a)* Find an initial B.F.S to the following transportation problem using Vogels Approximation Method and find an optimal solution 10

	Stores				Available
	S_1	S_2	S_3	S_4	↓
Warehouse A	5	1	3	3	34
Warehouse B	3	3	5	4	15
Warehouse C	6	4	4	3	12
Warehouse D	4	1	4	2	19
Requirement →	21	25	17	17	80

(b) The network gives the distances in miles between pairs of cities 1, 2, 3, ... and 7. Use Dijkstra's algorithm to find the shortest route between the following cities. 10

(i) cities 1 and 7

(ii) cities 3 and 6



6. *(a)* Determine the optimal transportation plan from the following table giving the plant to market shipping costs and quantities required at each market and available at each plan 10

	Market			
Plant	*A*	*B*	*C*	Available at plant
X	11	21	16	14
Y	7	17	13	26
Z	11	23	21	36
Required at market	18	28	25	

(b) A project consists of eight activities with the following relevant information

Activity	Immediate Predecessor	Estimated duration (days)		
		t_o	t_m	t_p
A	–	1	1	7
B	–	1	4	7
C	–	2	2	8
D	A	1	1	1
E	B	2	5	14
F	C	2	5	8
G	D,E	3	6	15
H	F,G	1	2	3

(i) Draw the PERT network and find out the expected Project completion.

(ii) What duration will have 95% confidence for Project completion ?

8+2=10

7. *(a)* The following are the details of estimated time of activities of a certain Project.

Activity	Immediate Predecessor	Time (days)
1-2	–	5
1-3	–	4
2-4	1-2	6
3-4	1-3	2
4-5	2-4	1
4-6	2-4 & 3-4	7
5-7	4-5	8
6-7	4-6	4
7-8	6-7 & 5-7	5

(i) Draw the network diagram.

(ii) Identify critical path and find the total Project duration.

(iii) Draw the time scaled diagram.

2+5+3=10

(b) Using duality theory solve the L.P.P

Maximize $Z = 2x_1 + x_2$

Subject to $-x_1 + 2x_2 \leq 2$

$x_1 + x_2 \leq 4$

$x_1 \leq 3$

and $x_1, x_2 \geq 0$ 10
