53 (IE 403) LSLG

2016

LINEAR SYSTEMS AND SIGNALS

Paper : IE 403

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Determine the fundamental period of the following signals if periodic 2×2=4

(i) $2\sin 200\pi t + 3\cos 4t$

(ii) $\cos(0{\cdot}03\pi n)$

(b) Calculate the energy or power of the signals : 2×2=4

(i) $x(t) = 5\sin(20t + \pi/3) + 7\cos(30t + \pi/2)$

(ii) $x(t) = u(t)$

(c) Draw the following signals : 2×2=4

(i) $u(-t+2)$

(ii) $\pi(t-3).u(t+4)$

(d) Check whether the system governed by equation $y(t) = 2x^2(t)$ is linear, time variant, causal and dynamic or not. 8

2. (a) Determine the convolution of the following signals : 2×4=8

(i) $x(t)=e^{-2t}u(t)\ ;\ y(t)= u(t+2)$

(ii) $x(t) = tu(t)\ ;\ y(t)= e^{-3t}u(t+1)$

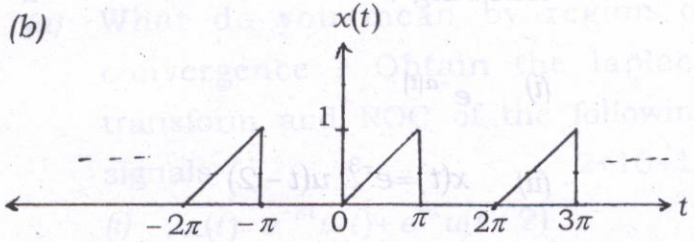
(b) Obtain the convolution of the following signals using graphical and matrix convolution methods 5+3=8

$$x_1[n]=[1,2,4,-2]$$
$$\uparrow$$
$$x_2[n]=[2,1,3,5]$$

(c) Derive the relationship between cross correlation function and convolution function. 4

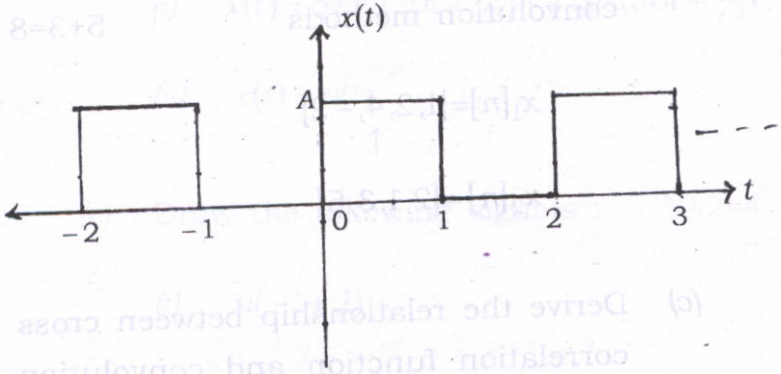
3. (a) Prove that $x(t-t_0) \xrightarrow{FS} C_n e^{-jnw_0 t}$ where C_n is the nth coefficient. 4

(b)



Determine the trigonometric fourier series for the waveform shown above. 8

(c) Obtain the exponential fourier series for the waveform shown below : 8



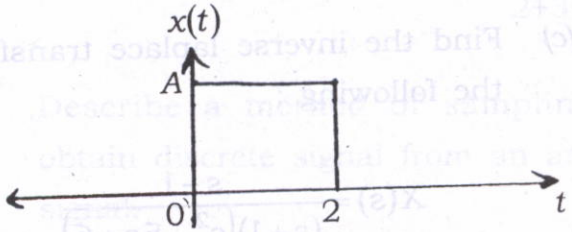
4. (a) State the Dirichlets condition for the existence of fourier transform. 2

(b) Find the fourier transform of the following : 2×4=8

(i) $e^{-a|t|}$

(ii) $x(t)=e^{-3t}\,u(t-2)$

(c) Obtain the magnitude spectrum of the following signal 4



(d) Find the inverse fourier transform of

$$X(w) = Ae^{-j\pi/2} \quad \text{for } 0 \le w \le w_0$$

$$= Ae^{-j\pi/2} \quad \text{for } -w_0 \le w \le 0 \qquad 6$$

5. (a) What do you mean by region of convergence ? Obtain the laplacc transform and ROC of the following signals 2+10=12

(i) $x(t)=e^{-3t}u(t)+e^{4t}u(t-2)$

(ii) $x(t)=e^{-at}\cos wt\,u(t)$

(b) State and prove the time scaling property of laplace transform. 3

(c) Find the inverse laplace transform of the following : 5

$$X(s)=\frac{s-1}{(s+1)\left(s^2+5s+6\right)}$$

6. (a) Prove that $x(n+m)\xleftrightarrow{z} z^m \times(z)$ 4

(b) Find the z-transform of the following : $2\times5=10$

(i) $x(n)=-b^n u(-n-1)+(0{\cdot}5)^n u(n)$

(ii) $x(n)=2^n \cos\frac{n\pi}{2}u(n)$

(c) Obtain the inverse z-transform of the following : 6

$$X(z)=\frac{z}{2z^2-3z+1}\ ;\ ROC\ |z|<½$$

7. *(a)* What is the function of anti aliasing filter ? Explain how data reconstruction takes place. Define sampling theorem. 2+3+1=6

(b) Describe a method of sampling to obtain discrete signal from an analog signal. 5

(c) Determine the Nyquist sampling rate and Nyquist sampling interval of the following : 3×3=9

(i) $x(t) = 2\,sinC(100\pi t)$

(ii) $rect\,300t$.

(iii) $x(t) = 1 + cos\,200\pi t + sin\,400\pi t$
