53 (IE 403) LSAS

2017

LINEAR SYSTEMS AND SIGNALS

Paper : IE 403

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Find the fundamental period and fundamental frequency of the following signals 2×3=6

   (i) $x(t) = e^{j\pi t} + e^{j3t}$

   (ii) $x[n] = \sin 2n + \cos 3n$

   (iii) $x(t) = 1 + U(t) + \sin 2t$

   (b) Determine the power and energy of the following signals 2×2=4

   (i) $x(t) = \sin 5\pi t + 1/2 \cos 2\pi t$

(ii) $x[n]=\left(\frac{1}{2}\right)^n u(n)$

(c) Evaluate the following 2×2=4

(i) $\int_{-\infty}^{\infty} x(t)\sin 5\pi t\ \delta(t)$

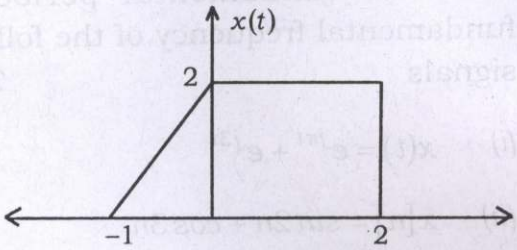
(ii) $\sum_{n=-5}^{3} 2n^2\,\delta(n+4)$

(d) Sketch the following signals 2×3=6

(i) $U(t)+2\pi(-t+3)$

(ii) $5\pi\left(t/2-3\right)+r(t-1)$

(iii) $3x(2t-1)$ if



2. (a) Determine whether the following system is dynamic, causal, linear and time variant or not 6

$$y[n]=2x[n]+\frac{1}{x[n-3]}$$

(b) Using graphical method, determine the output of the system if the input and impulse response of a system is given by $x(t) = U(t+2)$ and $h(t) = U(t-3)$.

6

(c) Determine $x_1(t) * x_2(t)$ if

$x_1(t) = sint\ u(t)$ and $x_2(t) = u(t-3)$

3

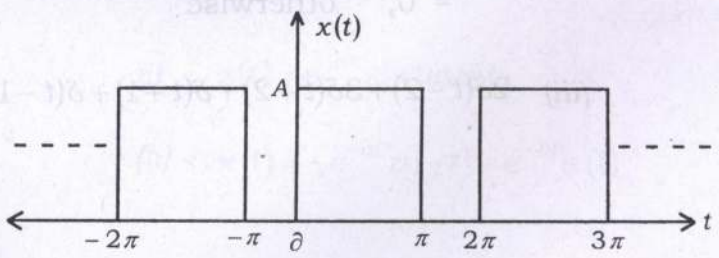
(d) What do you mean by cross correlation and auto correlation function ?

2+3=5

Prove that $R_{12}(\tau) = R_{21}^*(-\tau)$

3. (a) What are the conditions for the existence of Fourier series of a periodic signal ? 2+6=8

Obtain the Fourier series coefficient a_o, a_n and b_n of the following signals



(b) Prove that 2×3=6

(i) $x(t-2) \xrightarrow{FS} C_n e^{-jnw_0 2}$

(ii) $\frac{dx(t)}{dt} \xrightarrow{S} jnw_0 C_n$

(c) Find the exponential Fourier series for the signal $x(t)=t$, $0 \le t \le 1$, so that it repeats every 1 second. 6

4. (a) Prove the frequency shifting property of Fourier transform. 3

(b) Find the Fourier transform of 3+4+3=10

(i) $e^{-at}u(-t)$

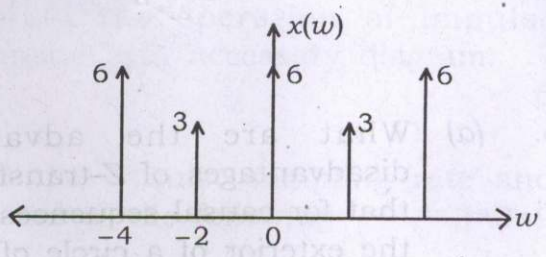
(ii) $x(t)=e^{-|t|}$ for $-2 \le t \le 2$
$= 0$, otherwise

(iii) $2\delta(t-2)+3\delta(t+2)+\delta(t+1)+\delta(t-1)$

(c) Find the inverse Fourier transform of 4+3=7

(i) $X(w) = \dfrac{4jw+6}{(ju)^2 + 6jw+8}$

(ii)



5. (a) What are the necessary conditions for the existence of Laplace transform ? Enumerate the advantages of Laplace transform ? 2+1+3=6

Derive the relationship between Laplace transform and Fourier transform.

(b) Obtain the Laplace transform of 2×4=8

(i) $x(t) = e^{-at} \sin wt\, u(t)$

(ii) $x(t) = -e^{-at} u(-t) + e^{-bt} u(t)$

(c) What do you mean by unilateral and bilateral Laplace transform ?

Determine the inverse Laplace transform of $X(s) = \dfrac{s+1}{s^3+4s^2+6s+4}$

1+5=6

6. (a) What are the advantage and disadvantages of Z-transform ? Prove that for causal sequences, the ROC is the exterior of a circle of radius r.

2+3=5

(b) Find the Z-transform of the following :

5+3+2=10

(i) $x(n) = 3(2/5)^n u(n) + 2(-1/3)^n u(n-1)$

(ii) $x(n) = a^{-n} u(-n-1)$

(iii) $x(n) = \{2, \underset{\uparrow}{1}, 3, 2, 0, 5\}$

(c) Find the inverse Z-transform of

$$X(z) = \frac{1}{2 - 4z^{-1} + 2z^{-2}}, \quad ROC, \ |z| > 1$$

5

7. *(a)* Define sampling theorem ? What are the effects of under-sampling ? Explain the operation of data reconstruction. 1+2+2=5

(b) Explain the operation of impulse sampling with necessary diagram. 6

(c) Find the Nyquist sampling rate and internal of the following — 3×3=9

(i) $x(t) = \frac{\sin 400\pi t}{\pi t}$

(ii) $5\sin 10\pi t.\cos 20\pi t$

(iii) $3\,sinc^2(50\pi t)$