Total number of printed pages–7

53 (IE 403) LSAS

2014

LINEAR SYSTEMS AND SIGNALS

Paper : IE 403

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Find whether the following signals are periodic or not, if periodic, determine the fundamental period 2×3=6

   (i) $x(t) = 20\cos(10\pi t + \pi/6)$

   (ii) $x(n) = \sin(0 \cdot 2\pi n)$

   (iii) $x(n) = \frac{3}{5}\, e^{j.3\pi(n+1/2)}$

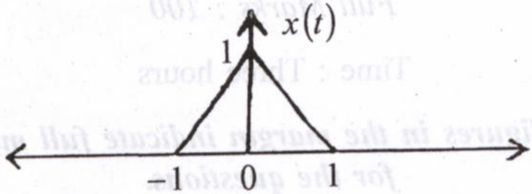
Contd.

(b) Determine the energy and power of the following signals 2×2=4

(i) $x(t)=e^{-3t}u(t)$

(ii) $x(t)=cos\,2t$

(c) The signal $x(t)$ is given below



Draw the sketch of the signal

$$y(t)=3x(2t+3)+x(t/2-1)$$ 2

(d) Check whether the following system is causal, linear, time invariant and memoryless

$$y(n)=nx(n)$$ 8

2. (a) Prove that 2+3=5

(i) $\int_{-\infty}^{\infty} x(t)\delta(t-t_0)\,dt=x(t_0)$

(ii) $\delta(at)=\frac{1}{|a|}\delta(t)$

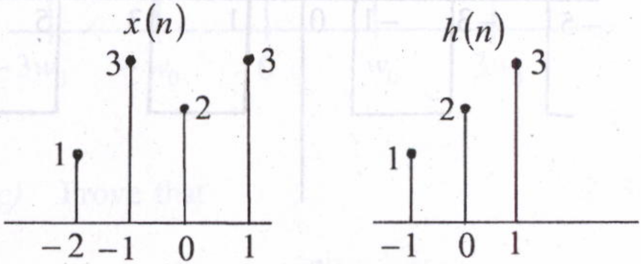
(b) Find the convolution of 3+2=5

(i) $x(t)=e^{-|t|};\ h(t)=e^{-2t}u(t+1)$

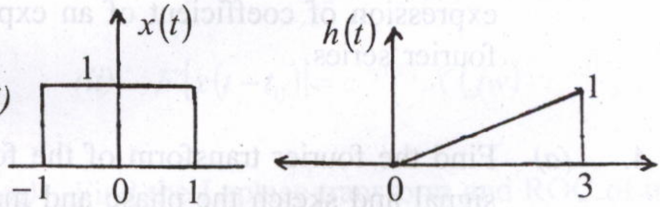
(ii) $x(t)=\sin t.u(t);\ h(t)=u(t)$

(c) Determine the convolution of the following signals using graphical method 5+5=10

(i)



(ii)

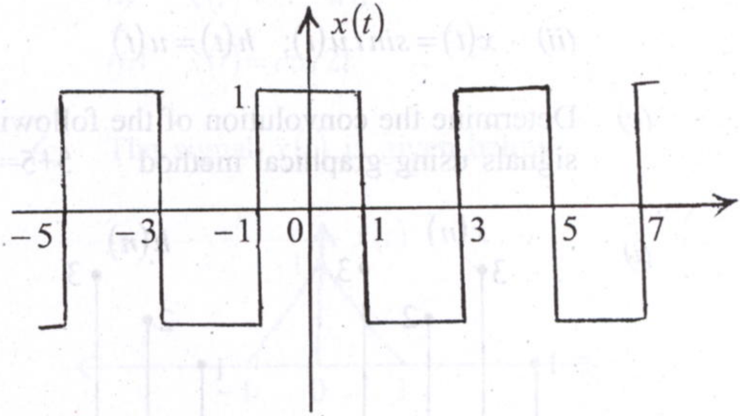


3. (a) If the fourier series (FS) coefficient of $x(t)$ is C_n, then prove that 3×2=6

(i) $FS[x(t-t_0)]=e^{-jn w_0 t_0}\cdot C_n$

(ii) $FS[x(-t)]=C_{-n}$

(b) Obtain the cosine fourier series representation for the following signals 7



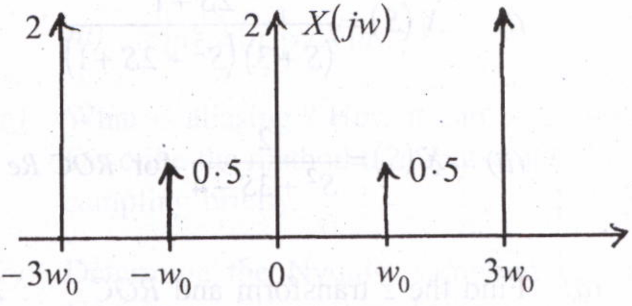
(c) State the Dirichlet's condition. Derive the expression of coefficient of an exponential fourier series. 2+5=7

4. (a) Find the fourier transform of the following signal and sketch the phase and magnitude spectrum 2×5=10

(i) $x(t) = e^{-3t}\left[u(t+2) - u(t-3)\right]$

(ii) $x(t) = e^{-2t}\,u(t)$

(b) Determine the inverse fourier transform of the following spectra 4



(c) Prove that 2×3=6

(i) $F\left[x(t)e^{-jw_0 t}\right] = X\left[j(w+w_0)\right]$

(ii) $F\left[x(t-t_0)\right] = e^{-jwt_0} X(jw)$

5. (a) Find the Laplace transform and ROC of the following 2×5=10

(i) $x(t) = e^{-b|t|}$

(ii) $x(t) = e^{-at}u(t) + e^{-bt}u(-t)$

(b) Find the inverse Laplace transform of 2×5=10

(i) $$X(S)=\frac{2S+1}{(S+3)\left(S^2+2S+1\right)}$$

(ii) $$X(S)=\frac{2}{S^2+3S-4} \text{ for } ROC\ Re(S)>1$$

6. (a) Find the z transform and ROC 2×4=8

(i) $$x(n)-\left(\frac{2}{3}\right)^n u(n)+\left(-\frac{1}{2}\right)^n u(n)$$

(ii) $$x(n)=sin\, w_0 n\ u(n)$$

(b) Find the inverse z transform of 2×6=12

(i) $$X(z)=\frac{1+3z^{-1}}{1+3z^{-1}+2z^{-2}},\ |z|>2$$

(ii) $$X(z)=\frac{1/4z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-1/4z^{-1}\right)},\ ROC\ |z|>1/2$$

7. *(a)* Prove that 2×3=6

(i) $z[x(n-m)] = z^{-m} X(z)$

(ii) $z[a^n x(n)] = X(a^{-1} z)$

(b) What is aliasing ? How it can be avoided ? Describe the method of impulse and flat top sampling briefly. 2+1+3=6

(c) Determine the Nyquist sampling rate and Nyquist sampling intervals for the following signals 2×4=8

(i) $sinc(100\pi t) + 2 sinc^2(80\pi t)$

(ii) $0 \cdot 5 sinc^2(50\pi t)$
