53 (IE 403) LSAS

2014

LINEAR SYSTEMS AND SIGNALS

Paper : IE 403

Full Marks : 100

Pass Marks : 30

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* Determine the fundamental period of the following signals if they are periodic 1½×2=3

   (i) $x[n] = e^{j(2\pi/3)n}$

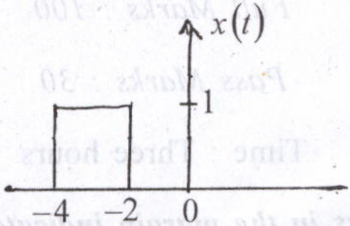
   (ii) $x(t) = 3\cos 4t + 2\sin(\pi t + 2)$

(b) Determine the power and energy of the following systems and conclude whether they are power and energy signals $2\times2=4$

(i) $x(t) = Ae^{jw_0 t}$

(ii) $x(t) = 10\cos(50t + \pi/3)$

(c) The signal $x(t)$ is shown below :



Draw the sketch of $y(t) = 2x(t/2) + x(2t-1)$ 3

(d) Show that the moving average system described by

$$y(n) = \frac{y}{3}\left[x[x] + x[n-1] + x[n-2]\right]$$ 2

is a BIBO stable.

(e) Determine whether the following signal is (i) Causal (ii) linear (iii) time invariant and (iv) memoryless. 8

$$y(n) = x(n) - nx(n-1)$$

2. *(a)* Evaluate the following : 2×3=6

(i) $\int_{0}^{5} \delta(t+3)\, e^{-t}\, dt$

(ii) $\int_{-\infty}^{\infty} (t-3)^2\, \delta(t-3)\, dt$

(iii) $e^{-at}u(t) * e^{-bt}u(t)$

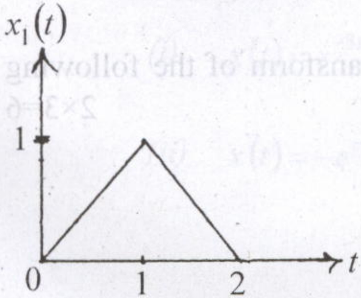
(b) Determine the convolution of the following signal using *(i)* graphical procedure and *(ii)* matrix convolution method 5+3=8

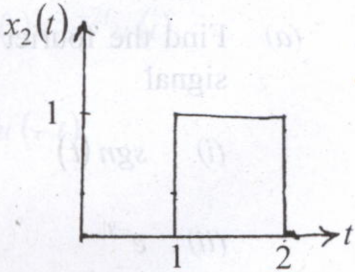
$$x_1[n] = [1, 3, 2, 5]$$

$$\uparrow$$

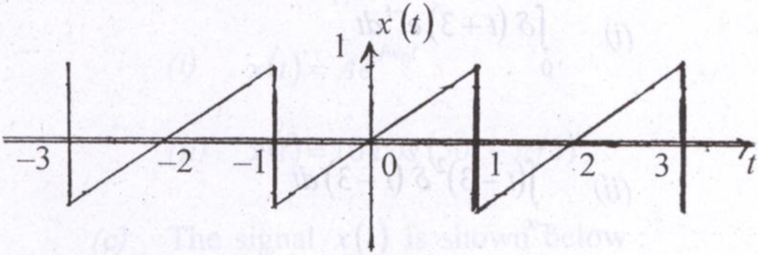
$$x_2[n] = [2, 4, 3, 7]$$

(c) Find the convolution of the following signals 6





3. *(a)* Find the trigonometric fourier series for the periodic signal shown below : 8



(b) If $x(t)$ is a periodic signal with fundamental period T and fourier series coefficients C_n, derive the fourier series coefficients of the following signals in term of C_n. 2×4=8

(i) $x(t+t_0)+x(t-t_0)$

(ii) $Re\{x(t)\}$

(c) Determine the fourier series representation for $x(t)=2\sin(2\pi t-3)+\sin 6\pi t$. 4

4. *(a)* Find the fourier transform of the following signal 2×3=6

(i) $sgn(t)$

(ii) $e^{-|t|}$

(b) If $F[x(t)] = X(jw)$ prove the following : 2×3=6

(i) $F[x(t-t_0)] = e^{-j\Omega t_0} X(jw)$

(ii) $F[x(at)] = \frac{1}{|a|} X(j\Omega/a)$

(c) Find the inverse fourier transform of the following : 2×4=8

(i) $X(jw) \cdot \frac{j\Omega}{(3+j\Omega)^2}$

(ii) $X(e^{jw}) = 1 + 2e^{-jw} + 2e^{-j2w} + 3e^{-j3w}$

5. (a) Find the Laplace transform and ROC of the following signals 2×5=10

(i) $x(t) = e^{-3t} u(t) + e^{-2t} u(t)$

(ii) $x(t) = -e^{-at} u(-t)$

(b) Find the inverse Laplace transform of the following : 2×5=10

(i) $X(s) = \frac{1+e^{-2s}}{3s^2+2s}$

(ii) $X(s) = \frac{3s^2+8s+6}{(s+2)(s^2+2s+1)}$

6. (a) State sampling theorem. Also discuss aliasing by taking example. 1+3=4

(b) The signal $x(t) = 10\,cos(10\pi t)$ is sampled at a rate of 8 sample per second. Plot the amplitude spectrum for $|\Omega| \le 30\,\pi$. Can the original signal be recovered from samplest ? Explain. 8

(c) Determine the Nyquist sampling rate and Nyquist sampling interval for the following signals

(i) $x(t) = sinc^2\,(200\pi t)$ 2×4=8

(ii) $x(t) = sinc\,(200\pi t) + 3\,sinc^2(120\pi t)$

7. *(a)* State and prove the Parseval's theorem using Z transform. 8

(b) Determine the Z-transform of the following signal 6

$$x(n) = \frac{1}{2}\left(n^2 + n\right)\left(\frac{1}{3}\right)^{n-1} u(n-1)$$

(c) Find the inverse Z-transform of the following : 6

$$X(Z) = \frac{1}{1 + 3Z^{-1} + 2Z^{-2}}, \ \text{ROC} \, |Z| > 2$$
