Total number of printed pages–7

53 (IE 403) LSAS

2012 C
2013
(May)

LINEAR SYSTEMS AND SIGNALS

Paper : IE 403

Full Marks : 100

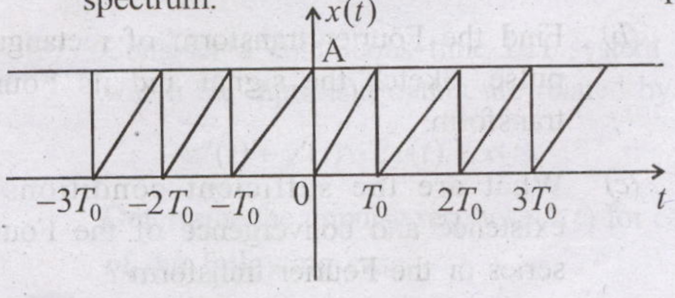
Pass Marks : 30

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Find the trigonometric Fourier series for the waveform shown below and draw its line spectrum. 10



Contd.

(b) Consider a rectified sine wave signal $x(t)$ defined by 10

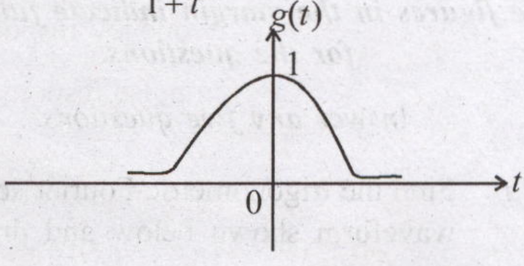
$$x(t) = |A \sin \pi t|$$

(i) Sketch $x(t)$ and find its fundamental period

(ii) Find the trigonometric Fourier Series of $x(t)$

2. (a) Find the Fourier transform $G(w)$ of the signal shown below : 7

$$g(t) = \frac{1}{1+t^2}$$



(b) Find the Fourier transform of rectangular pulse. Sketch the signal and its Fourier transform. 7

(c) What are the sufficient conditions to existence and convergence of the Fourier series or the Fourier transform ? 4

(d) Evaluate the following integral : 2

$$\int_{-1}^{1}\left(3t^2+1\right).\,\delta(t).\,dt$$

3\. (a) Obtain the conditions for the distortionless transmission of a signal through an LTI system. 2

(b) Consider a stable LTI system characterized by the differential equation 8

$$\frac{d^2y(t)}{dt^2}+4\frac{dy(t)}{dt}+3y(t)=\frac{dx(t)}{dt}+2x(t)$$

(i) Find the frequency response $H(w)$

(ii) Find the impulse response $h(t)$ of the system

(iii) What is the response of the system if the input is $e^{-t}.u(t)$?

(c) Consider a continuous–time LTI system for which the input and output are related by 6

$$y''(t)+y'(t)-2y(t)=x(t)$$

Determine the impulse response $h(t)$ for each of the following cases —

(i) The system is causal

(ii) The system is stable

(iii) The system is neither causal nor stable.

(d) Define the following : 2×2=4

(i) Signum function

(ii) Delta function.

4. (a) Prove the following convolution integral 3

$$x(t) * \delta(t + t_0) = x(t + t_0)$$

(b) Find the convolution of the two continuous time signals 7

$$f(t) = e^{-t^2} \text{ and } x(t) = 3t^2 \text{ ; for all } t.$$

(c) Sketch and explain the characteristics of an ideal low-pass, high-pass and band-pass filters. 6

(d) Explain the properties of correlation function. 4

Or

Bring out the relation between correlation and convolution.

5. *(a)* The signals 7

$g_1(t) = 10\,cos(100\pi t)$ and

$g_2(t) = 10\,cos(50\pi t)$

are both sampled with frequency $f_s = 75Hz$. Show that the two sequences of samples so obtained are identical.

(b) Determine the Nyquist rate corresponding to each of the following signals : 4

(i) $x(t) = 1 + cos(2000\pi t) + sin(4000\pi t)$

(ii) $x(t) = sin^2(200\pi t)$

(c) Consider the continuous–time signal

$$x(t) = cos(100\pi t)$$

Determine the minimum sampling rate required to avoid aliasing. 3

(d) State sampling theorem for low-pass signals. Define aliasing and flat-top sampling. 6

6. *(a)* Determine the Laplace Transform of 8

(i) $x(t) = -e^{-2t}.u(-t) + e^{-3t}.u(-t)$

(ii) $x(t) = e^{2t}.u(t) + e^{-3t}.u(-t)$

Sketch the pole-zero plot with the ROC in each case.

(b) Find the inverse Laplace Transform of

$$X(S) = \frac{-5S+7}{(S+2)(S+1)(S-1)}$$ 9

if the ROC is

(i) $R(S) > 1$

(ii) $R(S) < -2$

(iii) $-1 < R(S) < 1$

(c) State and prove linearity property of Laplace Transform. 3

7. (a) Determine the Z-transform of 8

$$x(n) = -(1/2)^m . u(-n-1) + 2^n . u(-n-1)$$

and depict the ROC and the location of poles and zeroes in the Z-plane.

(b) Using the power series expansion technique, determine the Inverse Z-transform of the following function 8

$$X(Z) = \frac{1+Z^{-1}}{1-(1/3)\,Z^{-1}}$$

when ROC : $|Z| > 1/3$

(c) Derive a relationship between Z-transform and discrete–time Fourier transform. 4
