53 (EC 603) DSPR

2017

DIGITAL SIGNAL PROCESSING

Paper : EC-603

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

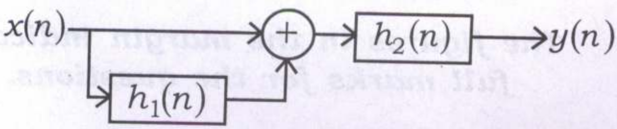
1. *(a)* Calculate the output sequence of the system with impulse response $h(n)=\left(\frac{1}{4}\right)^n u(n)$; when the input is a complex exponential sequence $x(n)=4\,exp\,(j\pi n/2)$; $-\infty<n<\infty$. Deduce the necessary theory. 6+4

(b) Find the z-transformation of

(i) $x(n)=2^n u(n-2)$ (ii) $x(n)=1$ 3+2

(c) Consider an LTI system whose frequency response is $H(e^{j\omega})=exp\left(-j\frac{\omega}{2}\right)$; $\omega<|\pi|$. Find whether or not the system is causal. Show your reasoning. 5

2. (a) Consider the system shown below :



where, $h_1(n)=\beta\,\delta(n-1)$ and $h_2(n)=\alpha^n u(n)$.

(i) Find the impulse response $h(n)$ of the system. 5

(ii) Find the frequency response of the overall system. 5

(iii) Specify a difference equation that relates the output $y(n)$ to the input $x(n)$. 5

(b) Show that if a discrete-time LPF is described by the difference equation

$$y(n) = -\sum_{k=1}^{N} ak\, y(n-k) + \sum_{k=0}^{M} bk\, x(n-K) \;;$$

then the discrete time filter described by

$$y(n) = -\sum_{k=1}^{N} (-1)^k ak\, y(n-k) + \sum_{k=0}^{M} (-1)^k bk\; x(n-K)$$

is a high-pass filter. 5

3. Show that the analog transfer function

$$Ha(S) = \frac{bS}{S^2 + bS + \Omega_0{}^2}; \; b > 0$$ has a band-pass magnitude response with $|Ha(j0)| = |Ha(j\infty)| = 0$ and $|Ha(j\Omega_0)| = 1$. Determine the frequencies Ω_1 and Ω_2 at which the gain to $3dB$ below the maximum value of $0dB$ at Ω_0. Show that $\Omega_0 = \sqrt{\Omega_1 \Omega_2}$. The difference $(\Omega_2 - \Omega_1)$ is called the $3\text{-}dB$ bandwidth of the bandpass transfer function. Hence show that $b = (\Omega_2 - \Omega_1)$.

7+3+5+5

4. *(a)* Design a second-order discrete-time Butterworth filter with cut off frequency of $1\,kHz$ and sampling frequency of 10^4 samples/sec using Bilinear transformation. 10

(b) Discuss the Decimation-In-Time (DIT) radix 2FFT algorithm. You may choose $N=8$. 10

5. *(a)* In the given problem, the characteristics of analog and digital implementation of a single-pole low-pass analog system will be reviewed :

$$Ha(S)=\frac{\alpha}{S+\alpha} \Rightarrow ha(t)=e^{-\alpha t}(\text{normalised})$$

(i) What is the gain at dc ? At what radian frequency is the analog frequency 3-dB down from its dc value ? At what frequency is the analog frequency response zero ? At what time has the analog impulse response decayed to $(1/e)$ of its initial value ? 5

(ii) 'Prewarp' the parameter 'α' and perform the bilinear transformation to obtain the digital system function '$H(Z)$' from the analog design. What is the gain at dc ? At what frequency (real-valued) is the response zero ? Give an expression for the 3-dB radian frequency. How many samples are there in the unit sample time domain response before it has decayed to $\left(1/e\right)$ of its initial value ? 10

(b) An ideal discrete-time HP filter with cut off frequency '$w_c = \pi/2$' was designed using the bilinear transformation with $T = 1\ ms$. What was the cut off frequency 'Ω_c' for the prototype continuous time ideal H-P filter ? 5

6. Write short notes on ***any two*** from the following : 10+10

(i) FIR system design using frequency sampling technique.

(ii) Digital resonator

(iii) Sub-band coding of speech signal

(iv) Analog and digital frequency.
