53 (EC 603) DSPR

2014

DIGITAL SIGNAL PROCESSING

Paper : EC 603

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* Consider a discrete-time LTI system whose input $x(n)$ and output $y(n)$ are related by $y(n) = \sum_{K=-\infty}^{n} 2^{K-n}\, x(K+1)$. Find whether the system is causal or not. 10

   (b) If a discrete-time LTI system is BIBO stable, show that the ROC of its system function $H(z)$ must contain the unit circle ; i.e., $|z| = 1$. 10

2. *(a)* Consider an LTI system whose frequency response is given by

$$H\left(e^{j\omega}\right)=\begin{cases} e^{-j\omega/2} ; & |\omega|\le\pi. \\ 0 ; & |\omega|>\pi. \end{cases}$$

Show with reasoning that whether the system is causal or non-causal. 10

(b) Consider a discrete-time LTI system whose input $x(n)$ and output $y(n)$ are related by $y(n)-ay(n-1)=x(n)$; where 'a' is any constant. Find $y(n)$ with the auxiliary condition $y(-1)=y-1$ and $x(n)=Kb^n u(n)$. 10

3. Find the impulse response $h(n)$ for each of the causal, discrete time LTI system satisfying the following difference equations and also indicate whether each system is FIR or IIR :

(a) $y(n)=x(n)-2x(n-2)+x(n-3)$

(b) $y(n)+2y(n-1)=x(n)+x(n-1)$

(c) $y(n)-0\cdot5y(n-2)=2x(n)-x(n-2)$. 20

4. *(a)* Prove that if $X+(z)\xleftrightarrow{z} x(n)$; then $x(\infty)=\underset{z\to1}{Lt}\,(z-1)X+(z)$. 10

(b) A causal discrete time LTI system is described by

$y(n)-\frac{3}{4}y(n-1)+\frac{1}{8}y(n-2)=x(n)$; where $x(n)$ & $y(n)$ are input and output of the system respectively.

(i) Find the system function $H(z)$.

(ii) Find the impulse response $h(n)$ of the system.

(iii) Find the step response $S(n)$ of the system. 10

5. A digital low-pass filter is to be designed with the following desired frequency response :

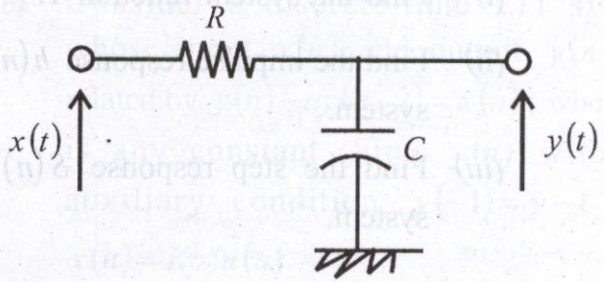
$$H_d(\omega)=\begin{cases} e^{-j2\omega}; & -\frac{\pi}{4}\le\omega\le\frac{\pi}{4} \\ 0\ ; & \text{elsewhere} \end{cases}$$

Determine the filter's coefficients $h_d(n)$ if the window function is defined as :

$$\omega(n)=\begin{cases} 1\ ; & 0\le n\le 4 \\ 0\ ; & \text{elsewhere.} \end{cases}$$

Also, find the frequency response $H(\omega)$ of the designed filter. 20

6. *(a)* For the given low-pass RC network $(R = 1M\Omega \ \& \ C = 1\mu F)$, determine the equivalent discrete-time expressions for the circuit response $y(n)$ when the input is $x(t) = e^{-2t}$ and the sampling frequency is $50Hz$. 15



(b) What is meant by : zero padding and LTI system ? 2½+2½

7. Give the necessary theory for calculating the output sequence from an LTI system with impulse response $h(n)$ when the input is a complex exponential sequence. Hence, find the output sequence of the system with the impulse response $h(n) = \left(\frac{1}{4}\right)^n u(n)$ when the input is a complex exponential sequence $x(n) = 4e^{j\frac{n\pi}{2}}$. 20