

## DIGITAL SIGNAL PROCESSING

Paper : EC 603

Full Marks : 100

Time : Three hours

**The figures in the margin indicate full marks for the questions.**

Answer **any five** questions.

1. (a) Distinguish between discrete-time signal and digital signal.
- (b) What is impulse response and what is its significance ?
- (c) How can you find the step response of a system if the impulse response is known ?
- (d) Compute  $y(n) = x(n) * h(n)$  ; where

$$x(n) = h(n) = \left\{ \underset{\uparrow}{1}, 2, -1 \right\}.$$

Contd.

- (e) Determine the impulse response of the following casual system

$$y(n) - 3y(n-1) + 2y(n-2) = x(n) + 3x(n-1) + 2x(n-2)$$

2+2+6+2+8

2. (a) Test the stability of the system :

$$y(n) = x(n) - x(-n-1) + x(n-1)$$

- (b) Given  $y(n) = x(n^2)$ , determine whether the system is : linear, time invariant, memoryless, causal.

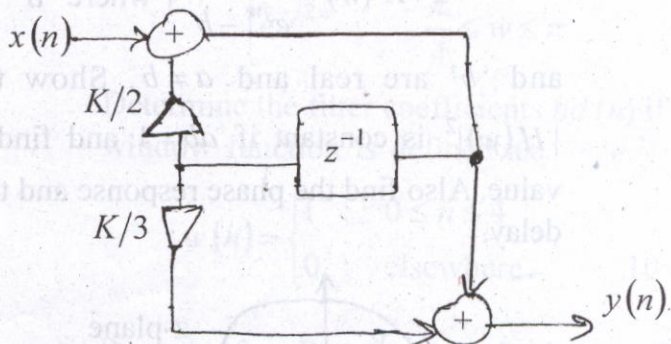
- (c) Find the frequency response of the system described by the system function

$$H(z) = \frac{1}{1 - 0.5z^{-1}} \quad 7+8+5$$

3. (a) The following transfer function is an FIR filter (N=11). Determine the magnitude response and show that the phase and group delays are constant

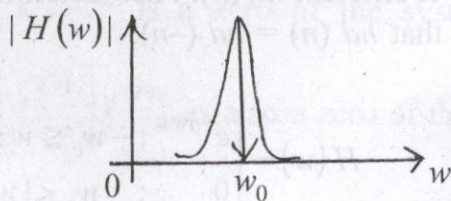
$$H(z) = \sum_{n=0}^{N-1} h(n)z^{-n}$$

- (b) Consider the discrete-time system shown below. For what values of  $K$  is the system BIBO stable?



10+10

4. (a) From the given pole-zero plot, determine the system transfer function and explain its filtering property.
- (b) What is a digital resonator? Show that the following frequency response can be achieved by a digital resonator.





(c) The frequency response of the system is given by

$$H(w) = \frac{e^{jw} - a}{e^{jw} - b} ; \text{ where 'a'}$$

and 'b' are real and  $a \neq b$ . Show that  $|H(w)|^2$  is constant if  $ab=1$  and find its value. Also find the phase response and time delay.

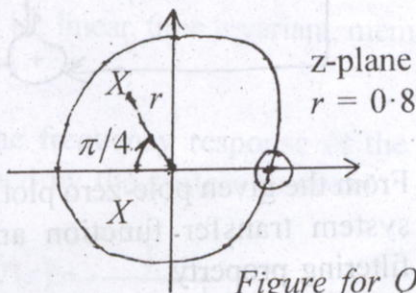


Figure for Q.4(a)

5+5+10

5. (a) A low pass filter should have the frequency response shown below. Find the filter coefficient  $hd(n)$ . Also determine ' $\tau$ ' such that  $hd(n) = hd(-n)$ .

$$H(w) = \begin{cases} e^{-jw\tau} & ; -w_c \leq w \leq w_c \\ 0 & ; w_c < |w| < \pi \end{cases}$$

- (b) A filter is to be designed with the following desired frequency response

$$H_d(w) = \begin{cases} 0 & ; \quad -\frac{\pi}{4} \leq w \leq \frac{\pi}{4} \\ e^{-j2w} & ; \quad \frac{\pi}{4} \leq w \leq \pi \end{cases}$$

Determine the filter coefficients  $hd(n)$  if the window function is defined as

$$w(h) = \begin{cases} 1 & ; \quad 0 \leq n \leq 4 \\ 0 & ; \quad \text{elsewhere.} \end{cases} \quad 10+10$$

6. (a) Prove that for a Butterworth analog lowpass filter

$$\Omega_C = \frac{\Omega_P}{\left(10^{0.1\alpha_P} - 1\right)^{\frac{1}{2N}}} = \frac{\Omega_S}{\left(10^{0.1\alpha_S} - 1\right)^{\frac{1}{2N}}}$$

where the symbols have their usual meaning.

- (b) An FIR LTI system has an impulse response  $h(n)$ , which is real valued, even and has finite duration of  $2N+1$ . Show that if

$z_1 = re^{jw_0}$  is a zero of the system, then

$z_1 = \left(\frac{1}{r}\right) e^{-jw_0}$  is also a zero of the system.

10+10