53 (EC 302) SISY

2017

LINEAR SYSTEMS & SIGNALS

Paper : EC-302

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions out of **seven**.*

1. *(a)* Determine the energy and power of the following signals : 3×4=12

   (i) $x_1(t)=e^{-t}$

   (ii) $x_2(t)=u(t)$

   (iii) $x_3(t)=e^{-t}u(t)$

(b) List and prove the properties of inner product in a complex vector space (CVS). For a CVS of functions, give the definition of inner product. 6+2=8

2. (a) What do you understand by mean square error ? Find the value of C if we approximate a complex valued function $f(t)$ in terms of another, $g(t)$, as $f_{approx}(t) = c\,g(t)$. 10

(b) What are orthonormal set of functions ? Give example to *any three* orthonormal functions defined over an interval [0, 1]. 10

3. (a) Find the trigonometric Fourier series expansion of a half wave rectifier output with peak voltage, V_m and period, 2π as shows is *Fig. 1*. 10

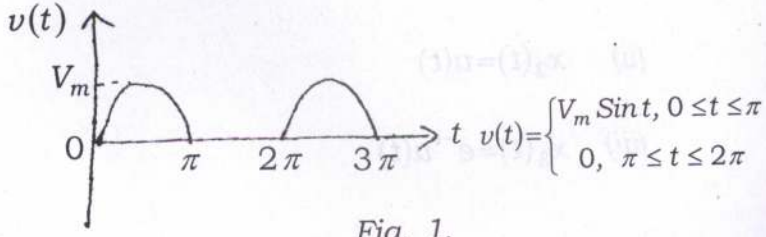

Fig. 1.

(b) State and prove Parseval's identity in the case of complex Fourier series representation. 5

(c) Distinguish between point-wise convergence and convergence in the mean in the Fourier synthesis of a periodic function. Explain how the type of covergence is related to Gibb's phenomena. 5

4. (a) Define impulse function. Show that the zero-state response of an LTI system can be written as the convolution of the input and the impulse response. 6

(b) Show that the energy spectral density and auto-correlation functions are Fourier transform pairs. 4

(c) Derive the Fourier transform of signum function from first principle. 5

(d) If $X(\omega)$ is the Fourier transform of $x(t)$, evaluate the Fourier transform of $(t-a)x(t-a)$. 5

5. *(a)* Discuss how one can synthesize an arbitrary discrete-time periodic signal $y(n)$ with period N using N-complex exponentials of the form $e^{j\Omega_k n}$; where $\Omega_k = k\frac{2\pi}{N}$ and k is an integer which takes values from 0 to $N-1$. 8

(b) Evaluate the Discrete-time Fourier transform (DTFT) of the signal $x(n) = \left(\frac{1}{2}\right)^n u(n)$. 4

(c) Discuss ideal low pass filter characteristics. Show that it does not satisfy Paley-Wiener criterion. 8

6. *(a)* Evaluate the Laplace transform of $f(t) = e^{-at}.\sin w_0 t.u(t)$. 6

(b) State and prove final value theorem. 4

(c) Consider a system represented by the differential equation

$$\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y(t)=-4x(t).$$ Using unilateral Laplace transform method, evaluate *(i)* zero-state response for the input $x(t)=e^{-t}u(t)$. *(ii)* zero-input response if $y(0^-)=-1$ and $\dot{y}(0^-)=5$. 10

7. (a) If $x(n)\leftrightarrow X(z)$ are Z-transform pairs, find the Z-transform of

(i) $n\,x(n)$, *(ii)* $\sum_{k=-\infty}^{n} x(k)$. 8

(b) If $x_1(n)\leftrightarrow X_1(z)$ and $x_2(n)\leftrightarrow X_2(z)$, evaluate the Z-transform of $x_1(n)*x_2(n)$. 4

(c) Find the inverse Z-transform of

$$X(z)=\frac{Z^2}{Z^2-\frac{3}{2}Z+\frac{1}{2}}$$ if the ROC is given as $0{\cdot}5<|z|<1$. 8
