53 (EC 302) SISY

2014

LINEAR SYSTEMS AND SIGNALS

Paper : EC 302

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Sketch the following : 2+3=5

   (i) $u(t+4)\,u(-t+4)$

   (ii) $r(t+2)-r(t+1)-r(t-1)+r(t-2)$

   (b) *(i)* Is the signal $y(t)=1+e^{j2\pi n/3}-e^{j4\pi n/7}$ periodic ? What is the period of $y(t)$? 2

(ii) Prove that power of the energy signal is zero over infinite time. 3

(c) Find even and odd components of the following signals : 4

(i) $1-2t+3t^3$

(ii) $\{2, 1, 4, 3, 5\}$
$\uparrow$

(d) (i) Check whether the following system is 4

1. Static or dynamic
2. Linear or non-linear
3. Causal or non-causal
4. Time invariant or time-variant

$$y(t)=at^2x(t)+bt\ x(t-4)$$

(ii) Test the causality and stability of the following system : 2

$$y(n)=x(n)-x(-n-1)+x(n-1)$$

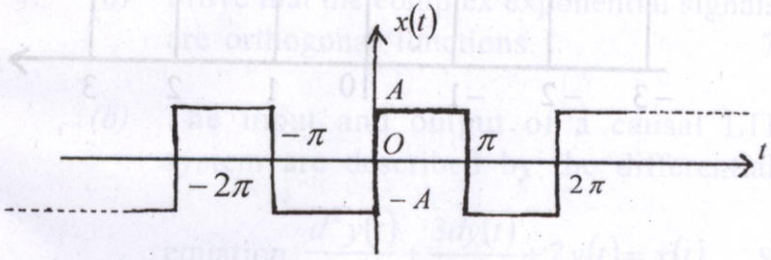
2. (a) State the condition for the Fourier series to exist. 4

(b) Prove the following properties of Fourier series : 4+4=8

(i) Time Reversal Property

(ii) Convolution Property

(c) Obtain exponential Fourier series of the following signal and also draw the frequency spectrum. 8



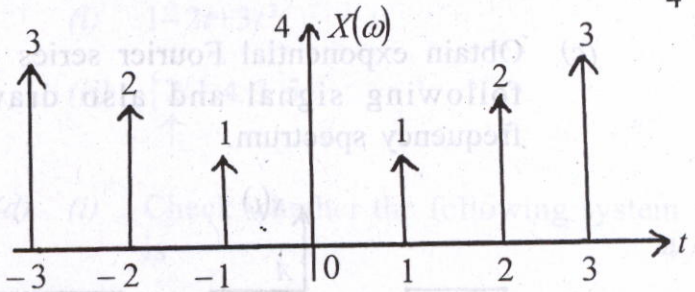
3. (a) Using properties of Fourier transform, find the Fourier transform of the following : 2+3=5

(i) $e^{-4t}u(t-8)$

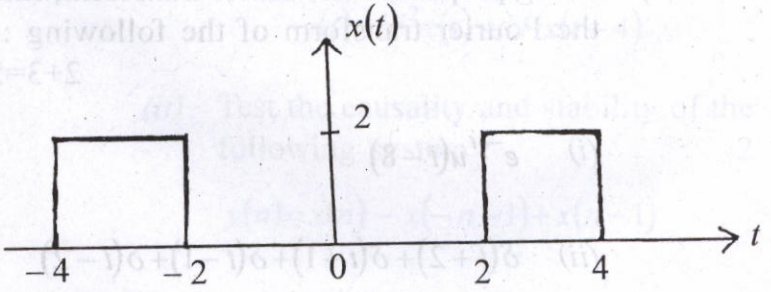
(ii) $\delta(t+2)+\delta(t+1)+\delta(t-1)+\delta(t-2)$

(b) State and prove Parseval's relation. 5

(c) Determine the inverse Fourier transform of the spectrum given in following figure : 4



(d) Find the Fourier transform of the signal shown in figure : 3



(e) A continuous time signal $x(t)$ has Fourier transform :

$$X(\omega)=\frac{\omega^3}{1+\omega^2}$$ 3

Determine the Fourier transform of signal

$$y(t)= x(1-t)+x(-1-t)$$

4. (a) Prove that the complex exponential signals are orthogonal functions. 7

(b) The input and output of a causal LTI system are described by the differential equation $\frac{d^2y(t)}{dt^2}+\frac{3dy(t)}{dt}+2y(t)=x(t)$. 8

(i) Find the frequency response of the system.

(ii) Find the impulse response of the system.

(iii) What is the response of the system if $x(t)=te^{-t}u(t)$?

(c) Consider a causal LTI system with frequency response $H(\omega)=1/(j\omega+3)$ for a particular input $x(t)$, this system is observed to produce the output $y(t)=e^{-t}u(t)-e^{-2t}u(t)$. Determine $x(t)$. 5

5. (a) Find the convolution of the signals $x_1(t)=t.u(t)\,;\, x_2(t)=tu(t)$ 3

(b) Find the convolution of the signals $x_1(t)=e^{-at}u(t)\,;\, x_2(t)=e^{-bt}u(t)$ using Fourier Transform. 3

(c) The functions to the system are given by

$$x(t)=\begin{cases}2 & \text{for } -2\le t\le 2\\ 0 & \text{elsewhere}\end{cases}$$

$$h(t)=\begin{cases}4 & \text{for } 0\le t\le 2\\ 0 & \text{elsewhere}\end{cases}$$ 7

Determine the output of the system graphically.

(d) Find the convolution of the following signals by graphical method 7

$x_1(t)=e^{-3t}u(t)$

$x_2(t)=u(t-3)-u(t-5)$

6. (a) State sampling theorem. Explain different types of sampling techniques. 7

(b) What is zero order hold ? Obtain the transfer function of zero order hold. 6

(c) Find the Nyquist rate and the Nyquist sampling interval for the following signals : 2+2=4

(i) $x(t)=\dfrac{sin(4000\pi t)}{\pi t}$

(ii) $x(t)=sinc(80\pi t)\,sinc(120\pi t)$

(d) The signal $x(t)$ with Fourier Transform $X(\omega)=u(\omega+\omega_0)-u(\omega-\omega_0)$ can undergo impulse sampling without aliasing provided that the sampling period $T<(\pi/\omega_0)$. Justify. 3

7. *(a)* Find the z-transform of $x(n)=n^2u(n)$. 4

(b) Determine the inverse z-transform of $X(Z)=\frac{1}{2-4z^{-1}+2z^{-2}}$ by long division method when *ROC* ; $|z|>1$ 4

(c) Determine all possible signals $x(n)$ associated with z-transform.

$$X(z)=\frac{(1/4)z^{-1}}{\left[1-(1/2)z^{-1}\right]\left[1-(1/4)z^{-1}\right]}$$ 7

(d) An *LTI* system is described by the difference equation 5

$$y(n)-\frac{9}{4}y(n-1)+\frac{1}{2}y(n-2)=x(n)-3x(n-1)$$

Specify the *ROC* of $H(z)$, and determine $h(n)$ for the following conditions :

(i) The system is stable

(ii) They system is causal.
