Total number of printed pages–7

53 (EC 302) SISY

2013

(December)

LINEAR SYSTEMS AND SIGNALS

Full Marks : 100

Pass Marks : 30

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

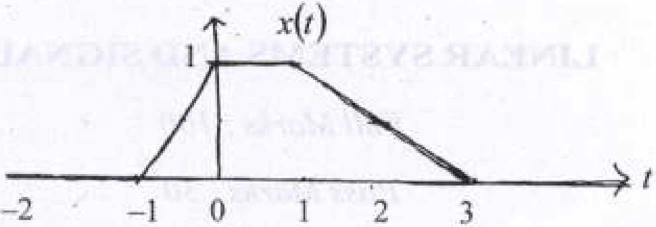
1. *(a)* Show that the complex exponential sequence $x(n)=e^{i\omega_0 n}$ is periodic only if $\frac{\omega_0}{2\pi}$ is a rational number. 2

   (b) Determine ever and odd component of following signals. 4

   (i) $x(t)=1+t\,tan\,t+t^2\,tan^2\,t$

   (ii) $x(t)=cos\left(\omega_0 t+\pi/3\right)$

(c) Determine energy in the signal $x\left(-\frac{1}{2}t+3\right)$, given the signal $x(t)$ as below 8



(d) Verify the following system for 6

(i) linearity

(ii) time invariance

(iii) casual

(iv) memoryless

$$y(t)=atx(t)+bt^2x(t-2)$$

2. (a) Define Fourier Series. Give Dirichlet conditions for fourier series. 5

(b) Find the exponential Fourier series coefficient of the signal

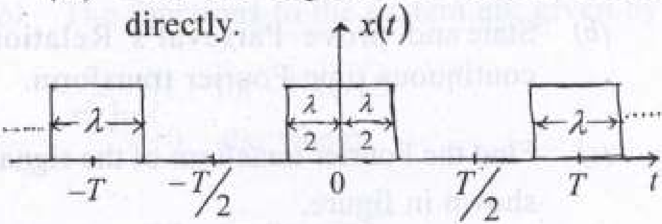
$$x(t)=2+cos(2\pi t/3)+4\,sin(5\pi t/3)$$ 5

(c) For the periodic gate function shown in figure. 10

(i) Find the trigonometric fourier series

(ii) Derive the corresponding exponential fourier series.

(iii) Find the exponential fourier series directly.

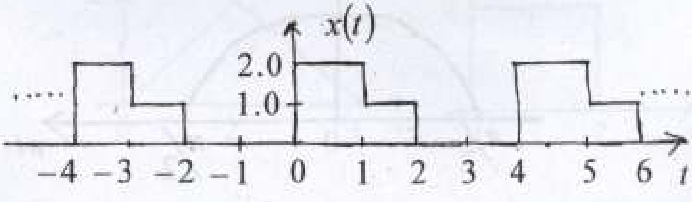


3. (a) What are orthogonal functions ? Prove that the functions $x_p(t)$ and $x_q(t)$ where,

$$x_r(t) = \frac{1}{\sqrt{T}}\left(\cos r\,\omega_0 t + \sin r\,\omega_0 t\right),$$

$T = (2\pi/\omega_0)$ are orthogonal over the period (0 to T). 10

(b) Compute the exponential Fourier series of the signal shown in the figure. 10



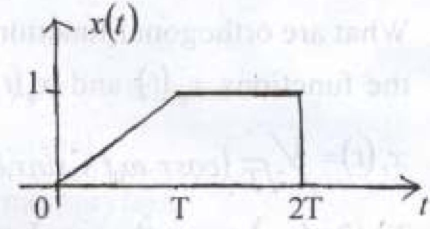
4. *(a)* Using properties of Fourier transform, find the fourier transform of the following :

(i) $e^{-3t}u(t-2)$ 4

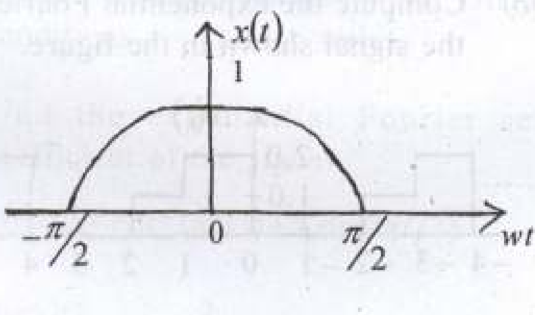
(ii) $u(-t)$

(b) State and prove Parseval's Relation for continuous time Fourier transform. 4

(c) Find the Fourier transform of the signal $x(t)$ shown in figure. 6



(d) Find the Fourier transform of the cosinusoidal pulse shown in figure. 6



5. (a) Find the convolution of the following signals : 3+3=6

(i) $x_1(t) = \cos t\, u(t);\quad x_2(t) = u(t)$

(ii) $x_1(t) = t.u(t)\ ;\quad x_2(t) = t\,u(t)$

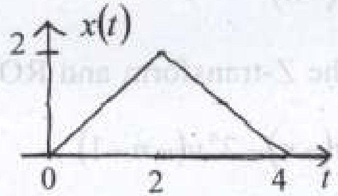
(b) The functions to the system are given by 7

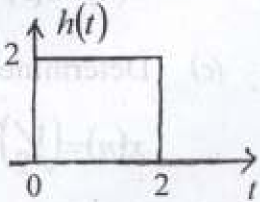
$$x(t) = \begin{cases} 1 & \text{for } -3 \le t \le 3 \\ 0 & \text{else where} \end{cases}$$

$$h(t) = \begin{cases} 2 & \text{for } 0 \le t \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

Determine the output of the system graphically.

(c) Find the convolution of the signals $x(t)$ and $h(t)$ shown in figure. 7





6. (a) State sampling theorem. Explain different types of sampling techniques. 7

(b) What is zero order hold ? Obtain the transfer function of zero order hold. 7

(c) Find the Nyquist rate and the Nyquist sampling interval for the following signals : 3+3=6

(i) $x(t) = -10 \sin 40\pi t \cos 300\pi t$

(ii) $x(t) = \text{sinc}\,(100\pi t) + 3\,\text{sinc}^2(60\pi t)$

7. (a) Find $x(0)$ if

$$X(z) = \frac{z+3}{(z+1)(z+2)}$$ 2

(b) Using appropriate properties, find the Z-transform of the signal

$x(n) = 2(3)^n u(-n)$ 5

(c) Determine the Z-transform and ROC of

$x(n) = \left(\frac{1}{2}\right)^n u(-n) - 2^n u(-n-1)$ 6

(d) Using long division, determine the inverse z-transform of 3

$$X(z)=\frac{z}{2z^2-3z+1};ROC\ ;|Z|>1$$

(e) Using partial fraction, find the inverse z-transform of

$$X(z)=\frac{3z^{-1}}{\left(1-z^{-1}\right)\left(1-2z^{-1}\right)}$$ 4

(i) if $ROC\ ;\ |z|>2$

(ii) if $ROC\ ;\ |z|<1$

(iii) if $ROC\ ;\ 1<|z|<2$
