53 (EC 302) LSYS

2019

LINEAR SYSTEMS AND SIGNALS

Paper : EC 302

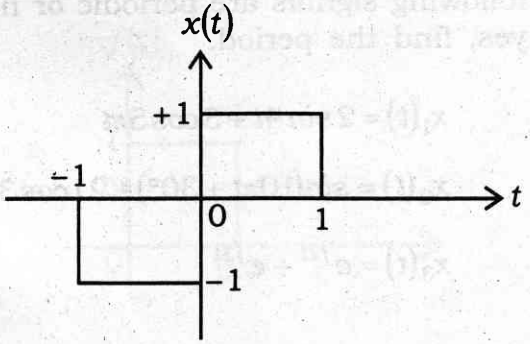
Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Define unit impulse function. Show that the derivative of unit step function obeys the sampling property of impulse function. Express the following function in terms of unit step function. 10



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(b) Given,

$$0.5x(-2t-1) = \begin{cases} \frac{t}{2}+1; & -2 \le t \le -1 \\ 2; & -1 \le t \le 1 \\ -t+2; & 1 \le t \le 2 \\ 0; & \text{otherwise} \end{cases}$$

Plot $x(t)$, $x(-2t)$, $x(t-1)$. 10

2. (a) Evaluate the energy and power of the following signals : 10

(i) $e^{-at}u(t);\ a>0$

(ii) $e^{j\omega t}$

(iii) $2u(t)-1$

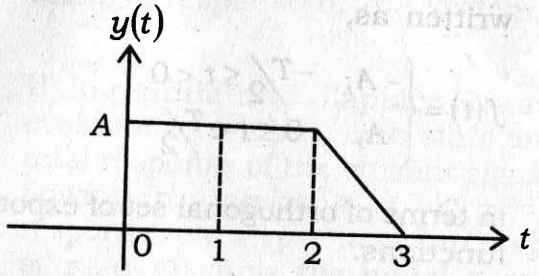
(b) Distinguish between periodic & non-periodic signals. Determine whether the following signals are periodic or not. If yes, find the period. 10

$$x_1(t) = 2\sin 4t + 3\cos 5\pi t$$

$$x_2(t) = \sin(10\pi t + 30°) + 21\cos 3\pi t$$

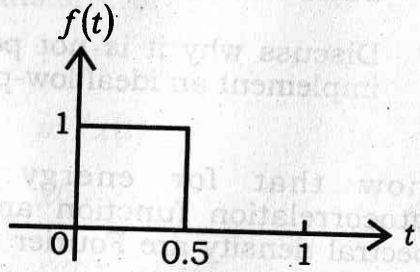
$$x_3(t) = e^{j2t} + e^{j3t}$$

3. (a) Find the even and odd part of the following signal and plot it. 5



(b) Define inner product and list its properties. 5

(c) Give example of an orthogonal set of 3 functions in the interval (0,1) and show that they satisfy the conditions for orthogonality. Approximate the following function $f(t)$ in terms of the above set. 10



4. *(a)* Derive the expression for exponential Fourier series coefficients. Expand the following function whose one period is written as,

$$f(t)=\begin{cases}-A, & -T/2 \le t < 0\\ A, & 0 \le t < T/2\end{cases}$$

in terms of orthogonal set of exponential functions. 10

(b) Evaluate the Fourier transform of the following signals : 10

(i) $x_1(t)=e^{-a|t|}; \quad a>0$

(ii) $x_2(t)=\begin{cases}A\left(1-\dfrac{|t|}{T}\right), & -T \le t \le T\\ 0, & \text{otherwise}\end{cases}$

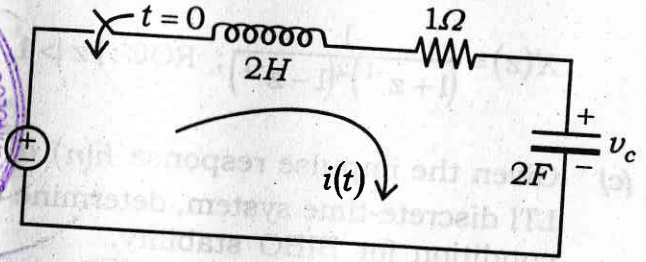
5. *(a)* *(i)* State and prove Nyquist theorem.

(ii) Discuss why it is not possible to implement an ideal low-pass filter. 10

(b) Show that for energy signals, autocorrelation function and energy spectral density are Fourier transform pairs. 5

(c) Show that for an LTI system, the zero state response is given by the convolution of input signal and the impulse response of the system. 5

6. *(a)* Using unilateral Laplace transform evaluate zero-input, zero-state and the total response of the current signal, $i(t)$ flowing in a series $R-L-C$ circuit in response to a step voltage input applied at $t=0$. Assume the initial charge on the capacitor as $v_c(0^-)=-1V$. 10



(b) If $x(t) \longleftrightarrow X(s)$ are the Laplace transform pairs, find the Laplace transform of 10

(i) $t\,x(t)$

(ii) $e^{-at}x(t)$

(iii) $x(t-\tau)$

(iv) $x(at); \quad a>0$

Contd.

7. *(a)* Find the Z-transform of the following sequences :

(i) $\cos(\omega n)\, u(n)$

(ii) $\left(\frac{1}{3}\right)^n [u(-n) - u(n-8)]$

and determine their ROCs. 10

(b) Find the inverse Z-transform of the following function, 6

$$X(z) = \frac{1}{(1+z^{-1})^2(1-z^{-1})}; \quad \text{ROC} : |z| > 1$$

(c) Given the impulse response $h(n)$ of an LTI discrete-time system, determine the condition for BIBO stability. 4