Total number of printed pages–7

53 (EC 302) LSSI

2018

LINEAR SYSTEMS AND SIGNALS

Paper : EC 302

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions out of **seven**.*

1. *(a)* Given a complex valued signal $f(t)$, describe how one can approximate it using a set of orthogonal functions, $\{g_i(t)\}$ where i varies from 1 to N. Derive the condition under which mean square error in the approximation is minimized. 10

Contd.

(b) For the signal (half wave rectifier output) given in *Fig. (1)*, evaluate the exponential Fourier series coefficients. 7

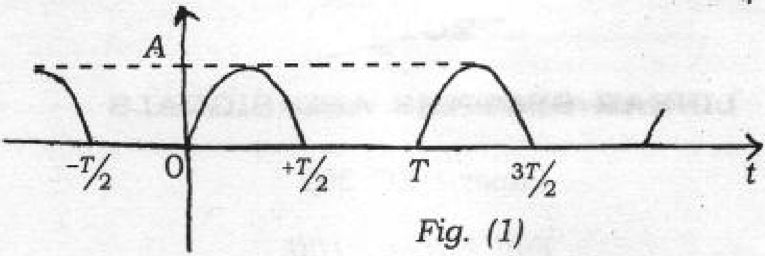

Fig. (1)

(c) List the conditions under which a periodic signal can be expressed in terms of Fourier series expansion. 3

2. (a) Define impulse function. Establish a relationship between unit impulse and unit step function. Show that any well behaved signal can be approximated in terms of shifted and scaled impulse function. 6

(b) Evaluate the energy and power of the signal, $f(t) = A.e^{j(\omega t+\theta)}u(t)$. 4

(c) Starting with Fourier Series synthesis and analysis expressions, derive the synthesis and analysis expressions of Fourier transform taking the appropriate limit. 5

(d) Derive the Fourier transform of signum function from first principles. Using this result evaluate the Fourier transform of unit-step function. 5

3. (a) If $x(t)$ has a Fourier transform, $X(\omega)$. Find the Fourier transform of the following : 10

(i) $e^{j\omega_0 t} x(t)$

(ii) $x(t-\tau)$

(iii) $x(at)$

(iv) $\int_{-\infty}^{t} x(t).\, dt$

(b) Find the Fourier transform of a triangular pulse function and plot its magnitude and phase spectrum. 5

(c) Explain what happens to the frequency spectrum of a band-limited signal when it is sampled in time using an impulse train, $\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$. 5

4. *(a)* Show that the zero-state output of a linear time-invariant system can be written as the convolution of the input signal and the impulse response. 5

(b) Check whether the following systems represents linear time-invariant systems

(i) $y(t) = x(-t)$

(ii) $y(t) = \frac{x(t)}{t}$

where $x(t)$ is the input to the system and $y(t)$ is the output. 5

(c) Evaluate the convolution and correlation between the signals, $x_1(t) = e^{-at}u(t)$ and $x_2(t) = e^{-bt}u(t)$. 6

(d) Evaluate and plot the even (symmetric) and odd (anti-symmetric) part of the signal, $x(t) = sin t.u(t)$. 4

5. *(a)* Explain what do you understand by distortionless transmission of a signal through the system. 4

(b) Check whether the systems represented by the following frequency responses are practically realizable or not ?

(i) $H(\omega) = e^{-\omega^2}$

(ii) $H(\omega) = rect\left(\frac{\omega}{2B}\right)$

where $2B$ is the width of the rectangular pulse function in frequency domain. 6

(c) Find the unilateral Laplace transform of the following signals : 10

(i) $x(t) = \sin\omega_0 t \;\; u(t)$

(ii) $x(t) = t^n \; u(t)$

(iii) $x(t) = e^{-a|t|} \cos\omega_0 t$

(iv) $x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nt)$

6. (a) Determine the initial and final values of $x(t)$ if its Laplace transform is given by $X(s) = \frac{10(2s+3)}{s(s^2+2s+5)}$. 5

(b) For a system represented by the transfer function, $H(s)=\frac{1}{s^2+5s+6}$; with $ROC: Re\{s\}>-2$, check whether it represent a causal and stable system. 5

(c) Determine the natural response of the system described by the following differential equation :

$$\frac{d^2y(t)}{dt^2}+5\frac{dy(t)}{dt}+6y(t)=x(t)$$

with initial conditions, $y(o^-)=-1$ and $\dot{y}(o^-)=5$. 5

(d) Evaluate the z-transform of $x(n)=n\ u(n)$. 5

7. (a) Determine the inverse z-transform of $X(z)=\frac{1}{1-1{\cdot}5z^{-1}+0{\cdot}5z^{-2}}$ if

(i) $ROC: |z|>1$

(ii) $ROC : |z| < 0{\cdot}5$

(iii) $ROC : 0{\cdot}5 < |z| < 1$

10

(b) Find the DTFT of the discrete-time unit step function, $u(n)$. 10
