2024

MATHEMATICS-III

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer any five questions

1. a) Using Bernoulli's theorem solve: **3x2=6**

(i) $\frac{dy}{dx}+\frac{y}{x}=\frac{y^2}{x^2}$ (ii) $\frac{dy}{dx}+\frac{y}{x}=x^2y^2$

b) Find the Orthogonal trajectories (any two): **4 x 2=8**

(i) $y=mx+c$ (ii) $xy=c$ (iii) $r^2=a^2\cos\theta$

c) Solve the following differential equation: **3 x 2 = 6**

(i) $(5x^4+3x^2y^2-2xy^3)dx+(2x^3y-3x^2y^2-5y^4)dy=0$

(ii) $(x+y-10)dx+(x-y-2)dy=0$

2. a) Solve the following homogenous differential equations: **4x4=16**

$(i)\ \frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=e^{3x}$ $(ii)\ \frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=x$

$(iii)\ (D^2+4)y=\sin 3x$ $(iv)(D^2-4D+4)y=x^3e^{2x}$

b) Using the method of variation of parameters solve $\frac{d^2y}{dx^2}+y=\mathrm{cosec}x$ **4**

3. a) Solve the following homogenous differential equations: **5 x 2 = 10**

(i) $(y^2-xy)dx+x^2dy=0$ (ii) $\frac{dy}{dx}+\frac{x-2y}{2x-y}=0$

b) A particle moves along the curve $x=4cost, y=4sint, z=6t$. Find the velocity and acceleration at time $t=0$. **5**

c) Find the unit tangent vector to any point on the curve $x=Acost, y=Asint, z=Bt$. **5**

4. a) For the matrices $A=\begin{bmatrix}2 & 1 & 3\\ 4 & 1 & 0\end{bmatrix}, B=\begin{bmatrix}1 & -1\\ 0 & 2\\ 5 & 0\end{bmatrix}$, verify that $(A.B)^T=B^T.A^T$. **5**

b) Express the matrix $A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 4 \\ 4 & 2 & 2 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix. **5**

c) Using elementary row operations find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}$. **10**

5. a) Find the adjoint and hence the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. **6**

b) Solve the following system of equations using the matrix inversion method: **7**

X + Y + Z = 6;   X + 2X = 7;   3X + Y + Z = 12

c) Find the rank of the following matrix by reducing it to the Echelon form: **7**

$$A = \begin{bmatrix} 2 & 1 & -1 & 4 \\ 1 & -1 & 2 & 12 \\ 2 & 2 & -1 & 9 \end{bmatrix}$$

6. a) If $\vec{r} = sint\hat{\imath} + cost\hat{\jmath} + t\hat{k}$, find $\frac{d\vec{r}}{dt}, \frac{d^2\vec{r}}{dt^2}, \left|\frac{d\vec{r}}{dt}\right|, \left|\frac{d^2\vec{r}}{dt^2}\right|$. **2.5x4=10**

b) Prove that (i) $div\vec{r} = 3$ (ii) $curl\vec{r} = 0$. **3+2=5**

c) Prove that a vector function $\vec{a}(t)$ to have constant magnitude iff $\vec{a}(t).\frac{d\vec{a}}{dt} = 0$. **5**

7. a) If $f(x,y,z) = 3x^2y - y^3z^2$ find $gradf$ at the point $(1,-2,-1)$. **5**

b) If $\vec{f} = x^2y\hat{\imath} - 2xz\hat{\jmath} + 2yz\hat{k}$, find $div\vec{f}$, $curl\vec{f}$, $curl(curl\vec{f})$). **5x3=15**

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