2024

MATHEMATICS III

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. a) For the matrices $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$ verify that: **5**

$$(A+B)^2 \neq A^2 + 2A.B + B^2$$

b) Find the adjoint of the matrix: $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ by any method. **5**

c) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 1 & 1 \end{bmatrix}$ by elementary row operations. **10**

2. **a)** For $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ 1 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$ verify whether A x B = B x A. **5**

b) Solve the following system of equations by matrix inversion:

$x + 2y + 3z = 10$; $2x - y + z = 5$; $3x + 2y - 5z = 8$ **8**

c) Find the rank of the following matrix by reducing it to the Echelon form: **7**

$$A = \begin{pmatrix} 1 & 4 & 5 & -9 & -7 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & 1 \\ 0 & -3 & -6 & 4 & 9 \end{pmatrix}$$

3. **a)** If $\vec{w} = 3t^3 i + cost\, j + 2t^2\, k$ then find the following:

(i) $\frac{d\vec{w}}{dt}$ (ii) $\left|\frac{d\vec{w}}{dt}\right|$ (iii) $\frac{d^2\vec{w}}{dt^2}$ **3**

b) Find the gradient of: $\emptyset(x,y,z) = x^3y + y^3z + z^3y + 2xyz.$ **3**

c) Find the divergence of:

$$\vec{F}(x,y,z) = xy^2z^3\, i + (2x^3y + z)j + y^3z^2\, k$$ **4**

d) Find the curl of: $\vec{F}(x,y,z) = xyz\, i + yz^2\, j + x^2y^2z\, k$ **5**

e) If $\vec{F}(x,y,z) = xy\,i + yz^2 j + x^2yz\,k$, then find *grad(div*$(\vec{F})$*)*. **5**

4. **a)** Find *ORDER* and *DEGREE* of the following differential equations:

(i) $dy = (x + sinx)dx$ (ii) $\left(\frac{dy}{dx}\right)^2 + 2y^2 = 4\frac{dy}{dx} + 4x$

(iii) $y = x\frac{dy}{dx} + r\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ (iv) $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + a^2y = 0$

2x4=8

b) Form differential equations from the following primitives:

(i) $y = Acosx + Bsinx$ (ii) $y = (A + Bx)e^{5x}$

where A, B are arbitrary constants.

6x2=12

5. Solve (any *four*):

(i) $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ (ii) $\frac{dy}{dx} = \frac{3x - 4y + 2}{4x - 5y + 3}$

(iii) $(x^2 + y^2 + 1)dx - 2xydy = 0$ (iv) $(1 - x^2)\frac{dy}{dx} - xy = 1$

(v) $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$

5x4=20

6. Solve (any *four*):

(i) $(D^2 + 2D + 5)y = 0$ (ii) $\frac{d^2y}{dx^2} + 4y = x^2$

(iii) $2\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + y = e^x + 1$ (iv) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = \sin x$

(v) $(D^2 - 9)y = xe^{3x}$

5x4=20

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