Total number of printed pages: 02 **D/3rd/DMA301**

2023

MATHEMATICS III

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. **a)** Find *ORDER* and *DEGREE* of the following differential equations: **2x5=10**

(a) $\left(\frac{dy}{dx}\right)^3 - 3y^2 = 4\frac{dy}{dx} + 4x$ **(b)** $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

(c) $(1+x^2)dx - (1+y^2)dy = 0$ **(d)** $y = x\frac{dy}{dx} + \frac{K}{\frac{dy}{dx}}$

(e) $\frac{d^3y}{dx^3} - y = 0$

b) Form differential equation whose solution is $y = Acosx + Bsinx$, where A and B are arbitrary constants. **5**

c) Solve (any one) : **(i)** $e^{x-y}dx + e^{y-x}dy = 0$ **(ii)** $\frac{dy}{dx} = -\frac{y^4+4x^3y+3x}{x^4+4xy^3+y+1}$ **5**

2. Solve: **(i)** $\frac{dy}{dx} = \frac{y^2-x^2}{2xy}$ **(ii)** $\frac{dy}{dx} = \frac{6x-4y+3}{3x-2y+1}$ **(iii)** $\frac{dy}{dx} + ysecx = tanx$ **5x4=20**

(iv) $(y^4 + 4x^3y + 3x)dx + (x^4 + 4xy^3 + y + 1)dy = 0$

3. Solve: **(i)** $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$ **(ii)** $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = x$ **5x4=20**

(iii) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = e^{3x}$ **(iv)** $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = sinx$

4. a) For $A=\begin{bmatrix}1 & 2 & 3\\ 0 & 2 & 1\\ 1 & 2 & 5\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0\\ 0 & 1\\ 2 & 1\end{bmatrix}$ verify whether A x B = B x A. **5**

b) Find the adjoint of the matrix: $A = \begin{bmatrix}1 & 2 & 3\\ 3 & 2 & 1\\ 2 & 1 & 3\end{bmatrix}$. **5**

c) Find the inverse of the matrix $A = \begin{bmatrix}1 & 2 & 3\\ 3 & -2 & 1\\ 4 & 1 & 1\end{bmatrix}$ by elementary row operations. **10**

5. **a)** For the matrices A = $\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$ verify that:

$(A+B)^2 \neq A^2 + 2A.B + B^2$ **5**

b) Solve the following system of equations by matrix inversion:

x + 2y + 3z = 10; 2x – y +z = 5; 3x + 2y – 5z = 8 **8**

c) Find the rank of the following matrix by reducing it to the Echelon form:

$$A = \begin{pmatrix} 1 & 4 & 5 & -9 & -7 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & 1 \\ 0 & -3 & -6 & 4 & 9 \end{pmatrix}$$ **7**

6. **a)** If $\vec{w} = 3t^2 i + cos2t\, j + 2t\, k$ then find the following:

(i) $\frac{d\vec{w}}{dt}$ (ii) $\left|\frac{d\vec{w}}{dt}\right|$ (iii) $\frac{d^2\vec{w}}{dt^2}$ **3**

b) Find the gradient of: $\emptyset(x,y,z) = x^3y + y^3z + z^3y + 2xyz.$ **3**

c) Find the divergence of:

$$\vec{F}(x,y,z) = xy^2z^4\, i + (2x^2y + z)j + y^3z^2\, k$$ **4**

d) Find the curl of: $\vec{F}(x,y,z) = xyz\, i + yz^2\, j + x^2y^2z\, k$ **5**

e) If $\vec{F}(x,y,z) = xy\, i + yz^2 j + x^2yz\, k$, then find *grad(div*($\vec{F}$)). **5**

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