Total number of printed pages: 02 **Programme (D)/3rd/DMA301**

2022

MATHEMATICS III

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. a) Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix. 3

b) Find the values of x, y, z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the condition $A^T A = I_3$. 4

c) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ by finding the Adjoint of A. 6

d) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 4 \\ 1 & 3 & 3 \\ 1 & 4 & 3 \end{bmatrix}$ using elementary row operations. 7

2. a) Prove that $\nabla \cdot (\nabla \times A) = 0$. 2

b) If $\varphi = x^2y^2z^2$ and $A = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then find the value of $\frac{\partial^3}{\partial x^2 \partial z}(\varphi A)$. 4

c) If $A = x^2z\,\hat{\imath} - 2y^2z\,\hat{\jmath} + xyz^2\,\hat{k}$, then find $\nabla \times (\nabla \times A)$ at the point (1, -1, -1). 6

d) Solve the following system of equations using the matrix method: 8

$x + 2y + 5z = 10;$
$x - y - z = -2;$
$2x + 3y - z = -11$

3. a) Find $\nabla\varphi$ if $\varphi = \frac{1}{r}$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. 5

b) If $\varphi = x^2yz^2 - 3xy^2z$, then find the directional derivative of φ at the point $(1, 1, 1)$ in the direction of $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$. 5

c) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$. 10

4. a) Write *Order* and *Degree* of the following differential equations: 2x4=8

(i) $\frac{dy}{dx} = \frac{\sqrt{1-x^2}}{\sqrt{2-y^2}}$

(ii) $\left(\frac{dy}{dx}\right)^2 + 2y^2 = 4\frac{dy}{dx} + 4x$

(iii) $y = \sqrt{x}\frac{dy}{dx} + \frac{k}{\frac{dy}{dx}}$

(iv) $(1-x^2)\frac{dy}{dx} - xy = 1$

b) Form differential equations from the following equations: 4x3=12

(i) $y = Ae^{2x} + Be^{-2x}$

(ii) $y = e^x(ACosx + BSinx)$

(iii) $y = ACosx + BSinx$

where A and B are arbitrary constants

5. Solve: 5x4=20

a) $x^2(y+1)dx + y^2(x-1)dy = 0$

b) $(x^2 + y^2)dx + 2xydy = 0$

c) $x(x^2 + y^2 - a^2)dx + y(x^2 - y^2 - b^2)dy = 0$

d) $(1-x^2)\frac{dy}{dx} - xy = 1$

6. Solve: 5x4=20

a) $\frac{d^3y}{dx^3} - 13\frac{dy}{dx} - 12y = 0$

b) $\frac{d^2y}{dx^2} + 4y = x^2$

c) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2e^{3x}$

d) $\frac{d^2y}{dx^2} + a^2y = Sinax$

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