Total No. of printed pages = 8

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19/3rd Sem/DMA301

2021

MATHEMATICS – III

Full Marks – 100

Time – Three hours

The figures in the margin indicate full marks for the questions.

- Question No. 1 is compulsory and answer any *four* from the rest questions.

1\. (a) Fill in the blanks : 1×5=5

(i) The inverse of $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ is ______.

(ii) If $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$ then $A^2 - 7A + I_2$ equals to ______.

(iii) If $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$ then adj A is ______.

[Turn over

(iv) If $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\end{bmatrix}$ then rank of A is ______.

(v) If $A=\begin{bmatrix}4 & -1 & -4\\ 3 & 0 & -4\\ 3 & -1 & -3\end{bmatrix}$ then A^2 is ______.

(b) Choose the correct options : 1×5=5

(i) The gradient of the scalar field $\varnothing(x,y,z) = xyz$ is

(a) xy i + yz j + zx k

(b) zx i + xy j + yz k

(c) yz i + zx j + xy k

(d) x i + y j + z k

(ii) Gradient of a scalar field is a

(a) Scalar quantity

(b) Vector quantity

(c) A constant

(d) May be any quantity

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(iii) Divergence of a vector field is a

(a) Scalar quantity

(b) Vector quantity

(c) A constant

(d) May be any quantity

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(iv) Curl of a vector field is a

(a) Scalar quantity

(b) Vector quantity

(c) A constant

(d) May be any quantity

(v) Two vectors $\vec{a}$ and $\vec{b}$ are such that $\vec{a}.\vec{b} = 0$, then the two vectors are

(a) Parallel to each other

(b) Perpendicular to each other

(c) Equal to each other

(d) None of the above

(c) Choose the correct options : 2×5=10

(i) Order and degree of the differential equation $y = x\frac{dy}{dx} + \frac{a}{\frac{dy}{dx}}$ are respectively

(a) 1 and 2

(b) 2 and 1

(c) 1 and 3

(d) None of these

(ii) The differential equation of the primitive $y = Ae^{2x} + Be^{-2x}$, where A and B are arbitrary constants is

(a) $\frac{d^2y}{dx^2} - y = 0$

(b) $\frac{d^2y}{dx^2} + y = 0$

(c) $\frac{d^2y}{dx^2} = 0$

(d) None of these

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(iii) The differential equation of the primitive $y = Ax + A^2$, where A is an arbitrary constant is

(a) $\frac{d^2y}{dx^2} - y = 0$

(b) $\frac{dy}{dx} + y = 0$

(c) $y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2$

(d) $\frac{dy}{dx} = 0$

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(iv) Solution of the differential equation $ydx + xdy = 0$ is

(a) $xy = c$

(b) $x+y = c$

(c) $x^2+y^2 = 0$

(d) None of these

(v) Roots of the auxiliary equation of the differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=0$ are

(a) 2, 3

(b) 3, 4

(c) 5, 6

(d) None of these.

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2. (a) If $\vec{r}=3ti+2t^2j+t^3k$; find $\frac{d\vec{r}}{dt}$ at $t=1$. 2

(b) Find the divergence of: 4

$\vec{F}(x,y,z)=xy^2z^4i+(2x^2y+z)j+y^3z^2k$.

(c) Find the curl of: 4

$\vec{F}(x,y,z)=3xyz^2i+y^2\sin zj+xe^{2z}k$.

(d) If $\vec{F}(x,y,z)=xyi+yz^2j+x^2yzk$, then find:

$\nabla(\nabla.\vec{F})$ 5

(e) If $\varnothing(x,y,z)=xyz$ is a scalar function, find Curl of Gradient of $\varnothing$. 5

3. (a) If $A=\begin{bmatrix}3 & 2\\ 4 & 1\end{bmatrix}, B=\begin{bmatrix}a & b\\ 3 & 5\end{bmatrix}$ and AB = BA, find a and b. Also, compute 3A + 5B. 6

(b) If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$ show that $A^2-4A-5I_3=0$. 7

(c) For the matrices $A=\begin{bmatrix}2 & 0\\ 0 & 3\\ 1 & -\end{bmatrix}, B=\begin{bmatrix}3 & 4\\ 2 & 1\end{bmatrix}$, verify that $(AB)^t=B^tA^t$. 7

4. (a) Solve the following system of equations by matrix method : 7

$x-y+z=4$

$2x+y-3z=0$

$x+y+z=2$

(b) Find the inverse of the following matrix by elementary transformations : 7

$$D=\begin{bmatrix}1 & 2 & 6\\ 2 & 5 & 15\\ 6 & 15 & 46\end{bmatrix}$$

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(c) Find the rank of the matrix $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$ by elementary transformations. 6

5. (a) Solve the following : 5×3=15

(i) $(x^2+y^2)dx + 2xydy = 0$

(ii) $(y^4+4x^3y+3x)dx + (x^4+4xy^3+y+1)dy = 0$

(iii) $(1-x^2)\frac{dy}{dx} - xy = 1$.

(b) Find the orthogonal trajectories of the family of the parabolas $y = ax^2$. 5

6. Solve the following : 5×4=20

(a) $\frac{d^2y}{dx^2} + 4y = x^2$

(b) $\frac{d^2y}{dx^2} - a^2y = e^{ax}$

(c) $\frac{d^2y}{dx^2} + a^2y = \sin ax$

(d) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0$.

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