Total No. of printed pages = 8

19/3rd Sem/DMA301

ENTRA

## 2021

## MATHEMATICS - III

Full Marks - 100

Time - Three hours

The figures in the margin indicate full marks for the questions.

- Question No. 1 is compulsory and answer any four from the rest questions.
- 1. (a) Fill in the blanks:

1×5=5

- (i) The inverse of  $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$  is \_\_\_\_\_.
- (ii) If  $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$  then  $A^2 7A + I_2$  equals to
- (iii) If  $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$  then adj A is \_\_\_\_\_

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(iv) If 
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$$
 then rank of A is

(v) If 
$$A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$
 then  $A^2$  is \_\_\_\_\_.

- (b) Choose the correct options: 1×5=5
  - (i) The gradient of the scalar field
     Ø(x,y,z) = xyz is

(a) 
$$xyi + yzj + zxk$$

(b) 
$$zx i + xy j + yz k$$

(c) 
$$yzi + zxj + xyk$$

(d) 
$$xi + yj + zk$$

- (ii) Gradient of a scalar field is a
  - (a) Scalar quantity
  - (b) Vector quantity
  - (c) A constant
  - (d) May be any quantity

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(2)

- (iii) Divergence of a vector field is a
  - (a) Scalar quantity
  - (b) Vector quantity
  - (c) A constant
  - (d) May be any quantity
- (iv) Curl of a vector field is a
  - (a) Scalar quantity
  - (b) Vector quantity
  - (c) A constant
  - (d) May be any quantity
- (v) Two vectors  $\vec{a}$  and  $\vec{b}$  are such that  $\vec{a}, \vec{b} = 0$ , then the two vectors are
  - (a) Parallel to each other
  - (b) Perpendicular to each other
  - (c) Equal to each other
  - (d) None of the above

- (c) Choose the correct options:
  - (i) Order and degree of the differential equation  $y = x \frac{dy}{dx} + \frac{a}{\frac{dy}{dx}}$  are respectively

 $2 \times 5 = 10$ 

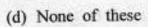
- (a) 1 and 2
- (b) 2 and 1
- (c) 1 and 3
- (d) None of these
- (ii) The differential equation of the primitive y = Ae<sup>2x</sup>+Be<sup>-2x</sup>, where A and B are arbitrary constants is

(4)

(a) 
$$\frac{d^2y}{dx^2} - y = 0$$

(b) 
$$\frac{d^2y}{dx^2} + y = 0$$

(c) 
$$\frac{d^2y}{dx^2} = 0$$





(iii) The differential equation of the primitive y = Ax+A², where A is an arbitrary constant is

$$(a) \quad \frac{d^2y}{dx^2} - y = 0$$

(b) 
$$\frac{dy}{dx} + y = 0$$



(c) 
$$y = x \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2$$

(d) 
$$\frac{dy}{dx} = 0$$

(iv) Solution of the differential equation ydx + xdy = 0 is

(a) 
$$xy = c$$

(b) 
$$x+y=c$$

(c) 
$$x^2+y^2=0$$

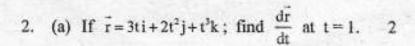
- (d) None of these
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- (5)

(v) Roots of the auxiliary equation of the differential equation  $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$  are





- (c) 5, 6
- (d) None of these.



- (b) Find the divergence of:  $\vec{F}(x,y,z) = xy^2z^4i + (2x^2y+z)j + y^3z^3k.$
- (c) Find the curl of: 4  $\vec{F}(x,y,z) = 3xyz^2i + y^2\sin zj + xe^{2z}k.$
- (d) If  $\vec{F}(x,y,z) = xyi + yz^2j + x^2yzk$ , then find:  $\nabla(\nabla \cdot \vec{F})$  5
- (e) If Ø(x,y,z)=xyz is a scalar function, find
   Curl of Gradient of Ø.

3. (a) If 
$$A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$$
,  $B = \begin{bmatrix} a & b \\ 3 & 5 \end{bmatrix}$  and  $AB = BA$ , find a and b. Also, compute  $3A + 5B$ .

(b) If 
$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$
 show that  $A^2 - 4A - 5I_3 = 0$ .

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(c) For the matrices 
$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \\ 1 & - \end{bmatrix}$$
,  $B = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$ ,

verify that (AB)t=BtAt.

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 (a) Solve the following system of equations by matrix method:

$$x - y + z = 4$$
$$2x + y - 3z = 0$$

$$x + y + z = 2$$

(b) Find the inverse of the following matrix by elementary transformations: 7

$$D = \begin{bmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{bmatrix}$$

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(7)

(c) Find the rank of the matrix 
$$\begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$$
 by

elementary transformations.

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(i) 
$$(x^2+y^2)dx + 2xydy = 0$$

(ii) 
$$(y^4+4x^3y+3x)dx + (x^4+4xy^3+y+1)dy = 0$$

(iii) 
$$(1-x^2)\frac{dy}{dx} - xy = 1$$
.

- (b) Find the orthogonal trajectories of the family of the parabolas y = ax².
  5
- 6. Solve the following:

5×4=20

(a) 
$$\frac{d^2y}{dx^2} + 4y = x^2$$

(b) 
$$\frac{d^2y}{dx^2} - a^2y = e^{ax}$$

(c) 
$$\frac{d^2y}{dx^2} + a^2y = \sin ax$$

(d) 
$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0$$
.

