

2023

MATHEMATICS-II

Full Marks: 100

Time: Three hours

*The figures in the margin indicate full marks for the questions.**Answer any five questions.*

1. a) Find the median and mode from the following: (5+5=10)
- | | | | | | |
|------------------|------|-------|-------|-------|-------|
| Class Interval : | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
| Frequency: | 4 | 10 | 6 | 20 | 10 |
- b) Calculate covariance and correlation co-efficient for the data which consists of the pairs: (3, 6), (0, 7), (4, 6) and (5, 1). (5+5=10)
2. a) The mean marks required by 25 students of section A of a class is 47, that of 35 students of section B is 51 and that of 30 students of section C is 53. Find the mean marks of the students of these three sections. (4)
- b) Find the standard deviation from the following: (6)
- | | | | | | |
|----|---|---|---|---|---|
| X: | 1 | 2 | 3 | 4 | 5 |
| F: | 2 | 1 | 8 | 2 | 4 |
- c) Find the equation of the straight line passing through the point (2, -3) and is perpendicular to the line $4x - 5y + 9 = 0$. (5)
- d) Find the acute angle between the lines $x - \sqrt{3}y = 1$ and $\sqrt{3}x - y = 4$. (5)
3. a) Find the equation of the circle passing through the points (1, 0), (0, 1) and (2, 1). (7)
- b) Show that the triangle having vertices A(a, 0), B(-a, 0) and C(0, $a\sqrt{3}$) is equilateral. (5)
- c) If the points (1, 0), (0, 1) and (a, b) are collinear, then prove that $a+b=1$. (4)
- d) Find the gradient and inclination of the line joining the points (3, 4) and (-2, -1). (4)

(2+2 = 4)

4. a) If $f(x) = x + 5$ and $g(x) = x^3 - 3$, then find

(i) $f(g(x))$ (ii) $g(f(x))$

- b) Evaluate the limit (any two)

(4 × 2 = 8)

(i) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$

(ii) $\lim_{x \rightarrow 1} \left\{ 2x + 5 \left(\frac{x^2}{x^2 + 1} \right) \right\}$

(iii) If $f(x) = \begin{cases} x^2 - 1, & \text{when } 0 < x < 1 \\ 2x - 1, & \text{when } 1 \leq x \leq 2 \end{cases}$

Find $\lim_{x \rightarrow 1} f(x)$

- c) If $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \begin{cases} 1, & \text{if } x \text{ belongs to } \mathbb{Z} \\ -1, & \text{if } x \text{ does not belong to } \mathbb{Z} \end{cases}$ (4)

Test the continuity of the above function at $x = 1$.

- d) Examine whether the following function is bijective or not (4)

$f: \mathbb{R} \rightarrow \mathbb{R}$

Defined by $f(x) = \frac{1}{1-x}$

5. a) Using definition, find the derivatives of the following functions (4+4=8)

(i) $f(x) = e^{2x}$

(ii) $f(x) = x$

- b) Evaluate (any three)

(3 × 2 = 9)

$\frac{dy}{dx}$ if

(i) $y = x^2 \log x$

(ii) $y = \frac{\sqrt{2x+3}}{dx}$ सत गमय तमसो मा ज्योतिर्गमय

(iii) $x = y^x$

(iv) $x^2 + y^2 = 1$

- c) Evaluate $\frac{d^2y}{dx^2}$ if $y^2 = 4ax$ (3)

6. a) Evaluate (any 2)

(2 × 5 = 10)

(i) $\int 4(7x - 2)^5 dx$

(ii) $\int \frac{\sin x + 2 \cos x}{2 \sin x + \cos x} dx$

(iii) $\int \cos^3 x dx$

(iv) $\int \frac{1}{x^2 + 4x - 1} dx$

- b) Evaluate (any two)

(5+5=10)

(i) $\int_2^4 2\sqrt{x} dx$

(ii) $\int_1^2 x \log x dx$

(iii) $\int_0^{\pi/2} \frac{1}{1 + \sin x} dx$
