***Total number of printed pages:* 2** **DIPLOMA/1st Semester (Back)/DMA103**

2024

MATHEMATICS-I

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

Answer any five questions.

1. a) Reduce $sin3A + cos\ A$ to a product of sine and cosine 4

   b) If $cosec\theta = \sqrt{2}$, find the values of $cot\ \theta$ and $sec\ \theta$. 4

   c) Find the maximum and minimum value of $3cos\theta + 4sin\theta$ 4

   d) If $cos\ B = 3/5$ and $sin\ A = 2/3$, then find $sin\ (A+B)$, where A is obtuse and B is acute angle. 4

   e) Evaluate $4sin^2 45^\circ + tan^2 60^\circ + cosec^2 30^\circ$ 4

2. a) Show that $sin135^\circ cos65^\circ + cos45^\circ cos115^\circ = 0$ 5

   b) The angle of elevation of the top of a tower is 30°. On walking 200 meters nearer, the elevation is found to be 60°. Find the height of the tower. 5

   c) Solve $2tan^2\emptyset + sec^2\emptyset = 2$, where $0 < \emptyset < 360^0$ 5

   d) Define Co-factor and Minor of an element. 2+3 =5

   If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & 5 \\ 8 & 3 \end{vmatrix}$, then find the value of x.

3. a) Find the unit vector in the direction of the sum of the vectors $2i + j + k$ and $i + 2j - k$. 5

   b) Find the conjugate of the complex number: $\frac{3-4i}{5+3i}$. 5

   c) If $\frac{1-iX}{1+iX} = A + iB$, show that $A^2 + B^2 = 1$. 5

   d) Find the modulus and argument of the complex number $2\sqrt{3} + 2i$. 5

4. a) A question paper is divided into two groups A and B each containing 5 questions. A candidate is required to answer 6 questions in all but he is not 5

permitted to attempt more than 4 questions from each group. Find the number of ways in which the candidate can choose his questions.

b) Find the number of triangles that can be formed by joining 10 points, 6 of which are in the same straight line. 5

c) Find the value of ***m*** for which the following vectors perpendicular: 5

$2\hat{\imath} + 3\hat{\jmath} - 4\hat{k};\ \hat{\imath} + 2\hat{\jmath} + m\hat{k}$

d) Find the angle between the vectors: $2\hat{\imath} + \hat{k}$ and $-\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ 5

5. a) Expand $\left(\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}}\right)^6$ 5

b) Find the term independent of x in the expansion of $\left(x^2 - \frac{1}{x^3}\right)^{20}$ 5

c) Simplify $log\frac{a^3b^3}{c^3} + log\frac{b^3c^3}{d^3} + log\frac{c^3d^3}{a^3} - 3logb^2c$ 5

d) Prove that: $7log\frac{10}{9} - 2log\frac{25}{24} + 3log\frac{81}{80} = log2$ 5

6 a) Given θ is an obtuse angle, and $\emptyset$ is acute angle. If $sin\theta = \frac{3}{5}$ and $tan\emptyset = \frac{5}{12}$, then evaluate $sin(\theta + \emptyset)$ 5

b) If the shadow of a vertical pole of 9 feet height standing on a horizontal plane is $\mathbf{3\sqrt{3}}$ feet, find the angle of elevation of the sun. 5

c) Prove that $\tan(A + B) = \frac{\tan A + \tan B}{1 - tanA\ tanB}$ 5

d) Find cos 4x in terms of cos x. 5

7 a) Show that: $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \cdots = e^{-1}$ 5

b) Show that: $\begin{vmatrix} sec\theta & sin\theta & tan\theta \\ 0 & 1 & 0 \\ tan\theta & cot\theta & sec\theta \end{vmatrix} = 1$ 5

c) Solve the following equations by Cramer's rule: 7

$$u + 2v + 3w = 6$$
$$2u + 4v + w = 7$$
$$3u + 2v + 9w = 14$$

d) Prove that: $\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\right) = 1$ 3

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