Total number of printed pages: 02 DIPLOMA/1ST Semester/DMA103

2023

MATHEMATICS-1

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

Answer any five questions.

1. a) Reduce $sin5A + cos\ A$ to a product of sine and cosine 4

   b) If $cosec\theta = \sqrt{2}$, find the values of $tan\ \theta$ and $sin\ \theta$. 4

   c) Find the maximum and minimum value of $7cos\theta + 24sin\theta$ 4

   d) Evaluate 4

   $$sin\ 420^{\circ} cos390^{\circ} + \cos(-300^{\circ})\sin(-330^{\circ})$$

   e) Prove that $cos20^{\circ}cos40^{\circ}\ cos60^{\circ}\ cos80^{\circ} = 1/16$ 4

2. a) Solve $cos\theta + \sqrt{3}sin\theta = 2$ where $0 < \theta < 360^{\circ}$ 5

   b) The angle of elevation of a stationary cloud from an aeroplane 200 meters above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°. Find the height of the cloud above the lake. 5

   c) Given that $cos\ 2A = ½$, then evaluate (i) $cot2A$ 2+3=5

   (ii) $cosec2A$

   d) Define Co-factor and Minor of an element. 2+3 =5

   If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & 5 \\ 8 & 3 \end{vmatrix}$, then find the value of x.

3. a) Express $\left(\frac{1+i}{1-i}\right)^3$ in the form of $a + ib$ and also find the complex conjugate. 5

   b) Find the real values of x and y if: $(x + iy)(2 + i) = 3 - i$. 5

   c) Find the modulus and argument of the complex number $\sqrt{3} - i$. 5

   d) Find the sum of the GP, $9, 3, 1, \ldots, \frac{1}{6561}$. 5

4. a) In a class test for 90 students, how many ways can the positions 1st, 2nd and 3

3rd be occupied?

b) In how many different ways can the letters of the word "MALAYALAM" be arranged amongst themselves. 2

c) From 6 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done to include at least two ladies? 3

d) Find the middle terms in the expansion of: $(x+\frac{1}{x})^{11}$. 4

e) Find the term independent of x in the expansion of $(x+\frac{1}{x})^{12}$ 4

f) Prove that: $log2+16log\frac{16}{15}+12log\frac{25}{24}+7log\frac{81}{80}=1$. 4

5. a) Prove that: $e=\frac{1}{1!}+\frac{2}{2!}+\frac{3}{3!}+\frac{4}{4!}+\cdots$ to infinity. 3

b) Find the vector in the direction of $i-j+k$ which has magnitude 7. 3

c) If $\vec{a}=5i-j-3k$ and $\vec{b}=i+3j-5k$ then show that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular to each other. 4

d) If $\Delta=\begin{vmatrix}1 & x & x^2\\ 1 & y & y^2\\ 1 & z & z^2\end{vmatrix}$ and $\Delta_1=\begin{vmatrix}1 & 1 & 1\\ yz & zx & xy\\ x & y & z\end{vmatrix}$ then prove that $\Delta+\Delta_1=0$. 5

e) Solve the system with three variables x, y and z by Cramer's Rule. 5

$x+2y+z=-5$

$3x+y-3z=4$

$-3x+4y+7z=7$

6 a) From the top of a lighthouse, the angles of depression of two ships are 30 and 45 degrees. The two ships, as it was observed from the top of the lighthouse, were 100 m apart. Find the height of the lighthouse. 5

b) A 80 m long ladder is leaning on a wall. If the ladder makes an angle of 45 degrees with the ground, find the distance of the ladder from the wall. 5

c) If sin A = 1 / √10 and sin B = 1 / √5, where A and B are positive acute angles, then what is A + B? 5

d) Find cos 4x in terms of cos x. 5