Total number of printed pages: **DIPLOMA/FIRST Semester/DMA103**

2022

MATHEMATICS-I

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1.	a)	If $sin\theta + sin^2\theta = 1$, then show that $cos^2\theta + cos^4\theta = 1$	4
	b)	If $cosec\theta = \sqrt{2}$, find the values of $\tan\theta$ and $\cos\theta$	4
	c)	Prove that $tan70^o = 2tan50^o + tan20^o$	4
	d)	Evaluate $4sin^2 45^\circ + cot^2 60^\circ + cosec^2 30^\circ$	4
	e)	Eliminate θ from the following equations $x = y(cosec\theta + cot\theta)$ and $z = y(cosec\theta - cot\theta)$	4
2.	a)	If A, B and C are the three positive acute angles. If $\sin(B + C - A) = 1$ $\cos(C + A - B) = 1$ and $\tan(A + B - C) = 1$, then find the value of A, B and C.	5
	b)	Solve $cos\theta + \sqrt{3}sin\theta = 1, 0^\circ < \theta < 360^\circ$	5
	c)	If $sinA = \frac{2}{3}$ and $cosB = \frac{3}{5}$, find the value of sin (A - B).	5
	d)	Find the maximum and minimum value of $7cos\theta + 24sin\theta$	5
3.	a)	In triangle ABC, show that $(b - c)cos\frac{A}{2} = asin\frac{B-C}{4}$, with usual meaning of the symbols.	5
	b)	The ratio of the angles of a triangle is 1 : 2 : 3 and the circum-radius (R) is 10 cm. Find the sides of the triangles and the area of that triangle.	5
	c)	A man standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°. When he retires 40 feet from the bank, he find that the angle to be 30°. Find the height of the tree and breadth of the river.	5

	d)	The angle of depression of two points on the ground, due East of a tower 100 feet height, observed from its top are 45° and 30° respectively. Calculate the distance between the objects.	5
4.	a)	If the sum of two unit vectors is a unit vector prove that the magnitude of their difference is $\sqrt{3}$.	5
	b)	Find the conjugate of the complex number: $\frac{3+4i}{5-3i}$.	5
	c)	If $\frac{1-ib}{1+ib} = x + iy$, show that $x^2 + y^2 = 1$.	5
	d)	Find the modulus and argument of the complex number $-2\sqrt{3} - 2i$.	5
5.	a)	A question paper is divided into two groups A and B each containing 5 questions. A candidate is required to answer 6 questions in all but he is not permitted to attempt more than 4 questions from each group. Find the number of ways in which the candidate can choose his questions.	5
	b)	Find the number of triangles that can be formed by joining 10 points, 6 of which are in the same straight line.	5
	c)	Find the value of ***m*** for which the following vectors perpendicular: $2\hat{\imath} + 3\hat{\jmath} - 4\hat{k};\ \hat{\imath} + 2\hat{\jmath} + m\hat{k}$	5
	d)	Find the angle between the vectors: $2\hat{\imath} + \hat{k}$ and $-\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$	5
6.	a)	Expand $\left(\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}}\right)^6$	5
	b)	Find the term independent of x in the expansion of $\left(x^2 - \frac{1}{x^3}\right)^{20}$	5
	c)	Simplify $log\frac{a^3b^3}{c^3} + log\frac{b^3c^3}{d^3} + log\frac{c^3d^3}{a^3} - 3logb^2c$	5
	d)	Prove that: $7log\frac{10}{9} - 2log\frac{25}{24} + 3log\frac{81}{80} = log2$	5
7.	a)	Show that: $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \cdots = e^{-1}$	5
	b)	Show that: $\begin{vmatrix} sec\theta & sin\theta & tan\theta \\ 0 & 1 & 0 \\ tan\theta & cot\theta & sec\theta \end{vmatrix} = 1$	5
	c)	Solve the following equations by Cramer's rule: $u + 2v + 3w = 6$; $2u + 4v + w = 7$; $3u + 2v + 9w = 14$	7
	d)	Prove that: $\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\right) = 1$	3