

Total number of printed pages:4

2021

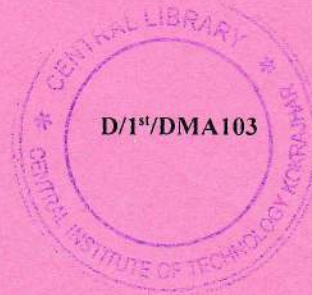
MATHEMATICS-I

Full Marks: 100

Time: Three hours

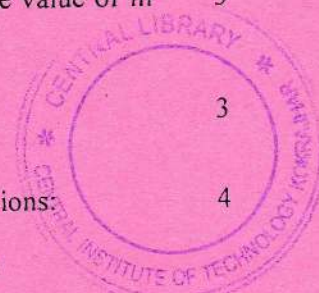
The figures in the margin indicate full marks for the questions.

Answer any five questions.



1. a) Write the true or false 1×10
- i) $\frac{\pi}{5}$ is a special angle of the trigonometric function.
 - ii) $\sin(\theta)$ is definable for all real value of θ .
 - iii) Maximum value of $\cos(\theta)$ is greater than 1.
 - iv) If $-1 \leq x \leq 1$, then $\cos^{-1}(-x) = -\cos^{-1}(x)$
 - v) $\sin^{-1}\left(\frac{2}{3}\right) + \cos^{-1}\left(\frac{8}{12}\right) = \frac{\pi}{2}$
 - vi) If \vec{a} and \vec{b} are two vectors such that $\vec{a} \cdot \vec{b} = 0$, they are perpendicular to each other.
 - vii) All possible combination of the digits 1, 2 and 3 is 7.
 - viii) The expansion $(x + x^2)^2$ has an Independent term.
 - ix) If a, b and c are in GP, then $b^2 = ac$
 - x) Conjugate of the complex number $\frac{1}{i}$ is i
- b) i) The sum of the first 3 terms in an A.P. is 51 and that of the last 3 terms is 99. If the A.P. has 11 terms, what is the value of the middle term? 4

- ii) The 5th term of a G.P. is -48 and the 7th term is -12 . Find the first term of the G.P. 3
- iii) Insert 4 geometric means between 3 and 729. 3
2. a) i) Express $\frac{(2-3i)^2}{2+3i}$ to the form of $A + iB$. 3
- ii) If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, then prove that $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$ 4
- iii) Express $\sqrt{3} + i$ into polar form. 3
- b) i) If $(m+n)P_2 = 56$, and $(m-n)P_2 = 12$, find the value of m and n . 3
- ii) Prove that, $nC_r + nC_{r-1} = n + 1C_r$ 3
- iii) Using Cramer's rule solve the equations: 4
- $$5x + 7y + 2 = 0$$
- $$4x + 6y + 3 = 0$$
3. a) i) Find the middle term in the following expansions 4
- $$\left(2x + \frac{1}{y}\right)^6$$
- $$\left(x - \frac{1}{x}\right)^9$$
- ii) Find the independent term of x in the expansion of $\left(2x + \frac{1}{3x^2}\right)^5$ 3
- iii) Find the co-efficient of x^4 in the expansion of $\left(x - \frac{1}{x}\right)^{11}$ 3
- b) i) Find the values of x , y and z so that the vectors 3



$\vec{a} = x\hat{i} + 2\hat{j} + z\hat{k}$ and $\vec{b} = 2\hat{i} + y\hat{j} + \hat{k}$ are equal.

- ii) Compute the magnitude of the following vectors: $\vec{a} = 2\hat{i} - 7\hat{j} - 3\hat{k}$, $\vec{b} = \frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$ 4
- iii) Find the sum of the vectors, $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$. 3
4. a) i) Find the unit vector in the direction of the vector: $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ 3
- ii) Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$ 3
- iii) Find the projection of the vector $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ on the vector $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ 4
- b) i) Write the value of $\sin(-660^\circ)$, $\tan(-\frac{31\pi}{3})$ 4
- ii) Find the value of $\cos(15^\circ)$ 3
- iii) Express as the sum of difference: $2\sin 3\theta \cos 4\theta$ and $2\sin \frac{\pi}{10} \sin \frac{3\pi}{4}$ 3
5. a) i) Find the principal value of $\cos^{-1}(-\frac{1}{2})$ 5
 $\sin\{\cos^{-1}(\frac{1}{2}) + \sin^{-1}(\frac{1}{2})\}$
- ii) Prove that $\cos \frac{\pi}{32} = \frac{1}{2} \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}$ 5
- b) i) Show that $\tan^{-1}(\frac{1}{2}) + \sin^{-1}(\frac{1}{\sqrt{10}}) = \frac{\pi}{4}$ 5
- ii) Find the modulus and argument of the complex number $\frac{1}{1+i}$ 5



6. a) i) Find the coordinates of the point $R(x,y)$ which divides the line segment joining the points $P(-4,7)$ and $Q(5,-9)$ in the ratio $3:2$ internally. 5
- ii) A Point moves in a plane such that the sum of its distances from the point $A(c,0)$ and $B(-c,0)$ is a constant $2a$. Prove that the equation to its locus is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $b^2 = a^2 - c^2$ 5
- b) i) Find the equation of the straight line passing through the point of intersection of the straight lines $x - 2y + 5 = 0$ and $2x - 5y + 6 = 0$, which also passes through the point $(-16,0)$ 5
- ii) Find the equation of the straight line passing through the point $(2,3)$ and parallel to the straight line joining the points $P(3,-4)$ and $Q(-5,6)$ 5

