Total number of printed pages:4

D/1st/DMA103

2021

MATHEMATICS-I

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

Answer any five questions.

1. a) Write the true or false 1×10

i) $\frac{\pi}{5}$ is a special angle of the trigonometric function.

ii) $\sin(\theta)$ is definable for all real value of θ.

iii) Maximum value of $\cos(\theta)$ is greater than 1.

iv) If $-1 \leq x \leq 1$, then $\cos^{-1}(-x) = -\cos^{-1}(x)$

v) $\sin^{-1}\left(\frac{2}{3}\right) + \cos^{-1}\left(\frac{8}{12}\right) = \frac{\pi}{2}$

vi) If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}.\vec{b} = 0$, they are perpendicular to each other.

vii) All possible combination of the digits 1, 2 and 3 is 7.

viii) The expansion $(x + x^2)^2$ has an Independent term.

ix) If a, b and c are in GP, then $b^2 = ac$

x) Conjugate of the complex number $\frac{1}{i}$ is i

b) i) The sum of the first 3 terms in an A.P. is 51 and that of the last 3 terms is 99. If the A.P. has 11 terms, what is the value of the middle term? 4

ii) The 5^{th} term of a G.P. is -48 and the 7^{th} term is -12. Find the first term of the G.P. 3

iii) Insert 4 geometric means between 3 and 729. 3

2. a) i) Express $\frac{(2-3i)^2}{2+3i}$ to the form of $A + iB$. 3

ii) If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, then prove that $(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$ 4

iii) Express $\sqrt{3}+i$ into polar form. 3

b) i) If $(m+n)_{P_2}=56$, and $(m-n)_{P_2}=12$, find the value of m and n. 3

ii) Prove that, ${}_nC_r + {}_nC_{r-1} = {}_{n+1}C_r$ 3

iii) Using Cramer's rule solve the equations: 4

$$5x + 7y + 2 = 0$$
$$4x + 6y + 3 = 0$$

3. a) i) Find the middle term in the following expansions 4

$\left(2x + \frac{1}{y}\right)^8$

$\left(x - \frac{1}{x}\right)^9$

ii) Find the independent term of x in the expansion of $\left(2x + \frac{1}{3x^2}\right)^5$ 3

iii) Find the co-efficient of x^4 in the expansion of $\left(x - \frac{1}{x}\right)^{11}$ 3

b) i) Find the values of x, y and z so that the vectors 3

$\vec{a} = x\hat{i} + 2\hat{j} + z\hat{k}$ and $\vec{b} = 2\hat{i} + y\hat{j} + \hat{k}$ are equal.

ii) Compute the magnitude of the following vectors: $\vec{a} = 2\hat{i} - 7\hat{j} - 3\hat{k}$, $\vec{b} = \frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$ 4

iii) Find the sum of the vectors, $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$. 3

4. a) i) Find the unit vector in the direction of the vector: $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ 3

ii) Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$ 3

iii) Find the projection of the vector $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ on the vector $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ 4

b) i) Write the value of $\sin(-660^\circ)$, $\tan(-\frac{31\pi}{3})$ 4

ii) Find the value of $\cos(15^\circ)$ 3

iii) Express as the sum of difference: $2\sin 3\theta\cos 4\theta$ and $2\sin\frac{\pi}{10}\sin\frac{3\pi}{4}$ 3

5. a) i) Find the principal value of 5

$\cos^{-1}(-\frac{1}{2})$

$\sin\left\{\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)\right\}$

ii) Prove that $\cos\frac{\pi}{32} = \frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}$ 5

b) i) Show that $\tan^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{\sqrt{10}}\right) = \frac{\pi}{4}$ 5

ii) Find the modulus and argument of the complex number $\frac{1}{1+i}$ 5

6. a) i) Find the coordinates of the point R(x,y) which divides the line segment joining the points P(-4,7) and Q(5,-9) in the ratio 3:2 internally. 5

ii) A Point moves in a plane such that the sum of its distances from the point A(c,0) and B(-c,0) is a constant 2a. Prove that the equation to its locus is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $b^2=a^2-c^2$ 5

b) i) Find the equation of the straight line passing through the point of intersection of the straight lines $x-2y+5=0$ and $2x-5y+6=0$, which also passes through the point (-16,0) 5

ii) Find the equation of the straight line passing through the point (2,3) and parallel to the straight line joining the points P(3,-4) and Q(-5,6) 5