CENTRAL INSTITUTE OF TECHNOLOGY KOKRAJHAR
(Deemed to be University)
KOKRAJHAR :: BTR :: ASSAM :: 783370

END – SEMESTER EXAMINATION
DIPLOMA

Session: July-December, 2024 Semester: 1st Time: 3 Hrs. Full Marks: 100

Course Code: DMA101 Course Title: Mathematics I

Answer Q.1 and ANY NINE from the rest

1.	(a)	Write the conditions for the two vectors: $a_1i + a_2j + a_3k$ and $b_1i + b_2j + b_3k$ to be parallel and perpendicular.	2
	(b)	What is the argument of the complex number lying in the: (i) Real axis, (ii) Imaginary axis.	2
	(c)	If a binomial expansion has even number of terms then the number of middle term(s) in the expansion is one. State TRUE or False	2
	(d)	The smallest positive integer n for which $(\frac{1+i}{1-i})^n = 1$, is _____	2
	(e)	For binomial coefficients: $C_0, C_1, C_2, \dots, C_n$, $C_0 + C_1 + C_2 + \cdots + C_n$=_____	2
2.	(a)	Find the vector in the direction of $2i + 3j + 3\sqrt{2}k$ which has magnitude 6.	3
	(b)	If $\vec{a} = 5i - j - 3k$ and $\vec{b} = i + 3j - 5k$ then show that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular to each other.	3
	(c)	Show that the points (1,2,7), (2,6,3) and (3,10, −1) are collinear.	4
3.	(a)	Express $(\frac{1+i}{1-i})^3$ in the form A + i B.	3
	(b)	Find real values of x and y for which the following equation is satisfied: $(x + iy).(2 + i) = 3 - i$	3
	(c)	Find the modulus and argument of: $1 - i\sqrt{3}$	4
4.	(a)	Find the middle term in the expansion of: $(x + \frac{1}{x})^8$	3
	(b)	Find the coefficient of x^6 in the expansion of $(x^3 + \frac{1}{x^3})^6$	3
	(c)	Find the term independent of x in the expansion of $(\frac{x}{2} - \frac{3}{x^2})^{18}$	4
5.	(a)	Find the value of the determinant: $A = \begin{vmatrix} x & sin\theta & cos\theta \\ -sin\theta & -x & 1 \\ cos\theta & 1 & x \end{vmatrix}$	4
	(b)	Solve the following system of equations by Cramer's rule: $5x - y + 4z = 5; 2x + 3y + 5z = 2; 5x - 2y + 6z = 1$	6
6.	(a)	Resolve into partial fractions (any one): (i) $\frac{6x^3+5x^2-7}{3x^2-2x-1}$, (ii) $\frac{2x+1}{3+2x-x^2}$, (iii) $\frac{2x+3}{6-5x+x^2}$	5
	(b)	Resolve into partial fractions (any one): (i) $\frac{x^2}{(x+1)^2(x+2)}$, (ii) $\frac{1-2x+2x^2}{(1-x)^3}$	5
7.	(a)	Evaluate the following limits (any one):	5

		(i) $lim_{x\to 1}\frac{2x^2-3x+1}{x^2-4x+3}$ (ii) $lim_{x\to\infty}\frac{x^3-2x^2+7x+6}{2x^3+4x+8}$ (iii) $lim_{x\to 2}\frac{3x^2-x-10}{x-2}$	
	(b)	Find the derivative from the first principle (any one): (i) x^3, (ii) $\sin x$	5
8.	(a)	If $cos\theta + cos^2\theta = 1$ show that $sin^2\theta + sin^4\theta = 1$	3
	(b)	If $\sin\theta + cosec\theta = 2$, then show that $sin^{16}\theta + cosec^{18}\theta = 2$	3
	(c)	If $tan\theta + sin\theta = m$ and $tan\theta - sin\theta = n$, then show that $m^2 - n^2 = 4\sqrt{mn}$	4
9.	(a)	Evaluate (any two): (i) $\tan(-1485^0)$, (ii) $\cos(495^0)$, (iii) $\sec(660^0)$	2+2
	(b)	Solve for θ lying between 0^0 and 360^0 (any one): (i) $\tan\theta = \frac{1}{\sqrt{3}}$, (ii) $sin^2\theta = \frac{3}{4}$	3
	(c)	If $\tan A = \frac{5}{6}$ and $\tan B = \frac{1}{11}$ then show that $A + B = \frac{\pi}{4}$	3
10	(a)	Show that $\tan 70^0 = 2\tan 50^0 + \tan 20^0$	3
	(b)	Prove that $\sin^2\left(\frac{\pi}{8}+\frac{A}{2}\right) - \sin^2\left(\frac{\pi}{8}-\frac{A}{2}\right) = \frac{1}{\sqrt{2}}\sin A$	4
	(c)	Prove that $\frac{\sin\theta+\sin 2\theta+\sin 4\theta+\sin 5\theta}{\cos\theta+\cos 2\theta+\cos 4\theta+\cos 5\theta} = \tan 3\theta$	3
11.	(a)	Show that $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$	5
	(b)	If $A + B + C = \pi$, then show that $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$	5
12.	(a)	In ΔABC, if $\sin^2 A + \sin^2 B = \sin^2 C$, then show that the triangle is right angled.	3
	(b)	In ΔABC, if $a\cos B = b\cos A$ show that the triangle is isosceles.	3
	(c)	In ΔABC, if $a\cos A = b\cos B$ show that the triangle is either isosceles or right angled.	4