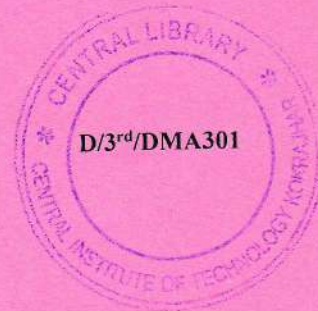


Total number of printed pages:4



2021

### MATHEMATICS-III

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

Answer Q. No. 1(compulsory) and any four from Q.No.2-7.

1. a) State True or False:

1x10=10

i) Vectors which are parallel to the same plane are coplanar vectors.

ii) If  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ , then  $r^2 = x^2 + y^2 + z^2$ .

iii) Two vectors  $\vec{a}$  and  $\vec{b}$  are orthogonal if  $\vec{a} \cdot \vec{b} = 0$ .

iv) A vector whose modulus is unity is called unit vector.

v)  $x^2 + y^2 = 1$  is an ordinary differential equation.

vi) Order of the differential equation  $\frac{dy}{dx} = x + \sin x$  is 1.

vii) Degree of the differential equation  $\frac{dy}{dx} = \frac{1+x^2}{1+y^2}$  is 2.

viii) Number of arbitrary constants in the solution of the differential equation  $\frac{d^2y}{dx^2} = y$  is 2.

ix) The differential equation  $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$  is homogeneous.

x) Differential equation of the form  $\frac{dy}{dx} + Py = Qy^n$ , where  $P$  and  $Q$  are functions of  $x$  or constants is called Bernoulli's differential equation.

b) Fill in the blanks:

1x10=10

i) The inverse of  $\begin{bmatrix} 4 & 3 \\ -7 & 1 \end{bmatrix}$  is -----

ii) If  $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$  then  $A^2 - 4A + 7I_2$  equals-----

iii) If  $A = \begin{bmatrix} 2 & -3 \\ 4 & 6 \end{bmatrix}$  then  $\text{adj } A$  is-----

iv) If  $A = \begin{bmatrix} 3 & -2 \\ -5 & 1 \end{bmatrix}$  then  $A - A^t$  is-----

v) If  $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$  then  $A^2$  is -----

vi) If  $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ , then  $A^2$  equals -----

vii) A vector is said to be solenoidal if  $\text{div. } f = \text{-----}$

viii) Curl of a vector point function is a -----quantity.

ix) If  $\vec{a} = \vec{i} + \vec{j}$ , then  $|\vec{r}^{-1}| = \text{-----}$

x) If  $\vec{a} = \vec{i} - \vec{j} + 3\vec{k}$ ,  $\vec{b} = 2\vec{j} - \vec{k}$ , then  $\vec{a} \cdot \vec{b} = \text{-----}$

2. Solve:

5 x 4=20

(i)  $(x^2 + y^2)dx + 2xydy = 0$

(ii)  $(y^4 + 4x^3y + 3x)dx + (x^4 + 4xy^3 + y + 1)dy = 0$

(iii)  $(\sin x \cos y + e^{2x})dx + (\cos x \sin y + \tan y)dy = 0$

(iv)  $(1 - x^2) \frac{dy}{dx} - xy = 1$

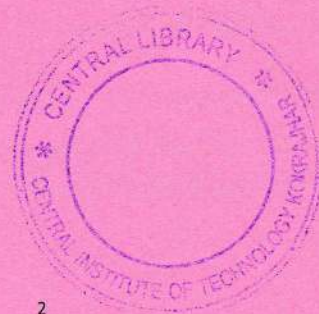
3. Solve (any four):

5 x 4=20

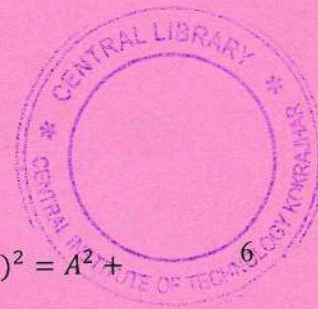
(i)  $\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = 0$

(ii)  $\frac{d^2y}{dx^2} + 4y = x^2$

(iii)  $\frac{d^2y}{dx^2} - a^2y = e^{ax}$







$$(iv) \frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = x$$

$$(v) \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2e^{3x}$$

4. a) If  $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ ,  $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$  and  $(A + B)^2 = A^2 + B^2$ , find  $a$  and  $b$ . 7

- b) If  $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$  show that  $A^3 - 23A - 40I_3 = 0$ . 7

- c) For the matrices  $A = \begin{bmatrix} 1 & -4 \\ 0 & 5 \\ 6 & 7 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & -7 \end{bmatrix}$  verify that  $(AB)^t = B^t A^t$ . 7

5. a) Solve the following system of equations by matrix method 7

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

- b) Find the inverse of the following matrix by elementary transformations. 7

$$D = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

- c) Find the rank of the matrix  $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$  by elementary transformations. 6

6. a) Show that the vectors  $2\vec{i} - \vec{j} + \vec{k}$ ,  $\vec{i} - 3\vec{j} - 5\vec{k}$  and  $3\vec{i} - 4\vec{j} - 4\vec{k}$  form a right angled triangle. 5

- b) Find the unit vector in the direction of  $2\vec{a} - \vec{b}$  where  $\vec{a} = 2\vec{i} - 3\vec{j} + 4\vec{k}$  and  $\vec{b} = \vec{i} - 2\vec{k}$ . 5

- c) If  $\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$  and  $\vec{b} = 5\vec{i} + \vec{j} + 2\vec{k}$  then find  $\vec{a} \cdot \vec{b}$  and  $\vec{a} \times \vec{b}$ . 2+3=5
- d) If  $f(x, y, z) = 3x^2y - y^3z^2$  find  $grad. f$  at the point  $(1, -2, -1)$ . 5
7. a) The position vectors of the points  $P$  and  $Q$  are  $2\vec{i} + 3\vec{j} - \vec{k}$  and  $4\vec{i} - 3\vec{j} + 2\vec{k}$  respectively. Determine  $\overrightarrow{PQ}$  and  $|\overrightarrow{PQ}|$ . 2+2=4
- b) Find the angle between the vectors  $2\vec{i} + 3\vec{j} - \vec{k}$  and  $\vec{i} - 2\vec{j} - 6\vec{k}$ . 5
- c) Find the area of a triangle having the points  $A(1,1,1)$ ,  $B(1,2,3)$  and  $C(2,3,1)$  as its vertices. 6
- d) If  $\vec{F} = x^2y\vec{i} + xz\vec{j} + 2yz\vec{k}$  then find  $curl\vec{F}$ . 5

