Total number of printed pages:4

D/3rd/DMA301

2021

MATHEMATICS-III

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

*Answer **Q. No. 1(compulsory) and any four from Q.No.2-7.***

1. a) State *True* or *False*: 1x10=10

i) Vectors which are parallel to the same plane are coplanar vectors.

ii) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then $r^2 = x^2 + y^2 + z^2$.

iii) Two vectors $\vec{a}$ and $\vec{b}$ are orthogonal if $\vec{a}.\vec{b} = 0$.

iv) A vector whose modulus is unity is called unit vector.

v) $x^2 + y^2 = 1$ is an ordinary differential equation.

vi) Order of the differential equation $\frac{dy}{dx} = x + sinx$ is 1.

vii) Degree of the differential equation $\frac{dy}{dx} = \frac{1+x^2}{1+y^2}$ is 2.

viii) Number of arbitrary constants in the solution of the differential equation $\frac{d^2y}{dx^2} = y$ is 2.

ix) The differential equation $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$ is homogeneous.

x) Differential equation of the form $\frac{dy}{dx} + Py = Qy^n$, where P and Q are functions of x or constants is called Bernoulli's differential equation.

b) Fill in the blanks: 1x10=10

i) The inverse of $\begin{bmatrix} 4 & 3 \\ -7 & 1 \end{bmatrix}$ is ----------

ii) If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ then $A^2 - 4A + 7I_2$ equals----------

iii) If $A = \begin{bmatrix} 2 & -3 \\ 4 & 6 \end{bmatrix}$ then $adj\ A$ is----------

iv) If $A = \begin{bmatrix} 3 & -2 \\ -5 & 1 \end{bmatrix}$ then $A - A^t$ is----------

v) If $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$ then A^2 is ----------

vi) If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$, then A^2 equals ----------

vii) A vector is said to be solenoidal if $div.\, f$ =-------

viii) Curl of a vector point function is a --------quantity.

ix) If $\vec{a} = \vec{\imath} + \vec{\jmath}$, then $|r^{\rightarrow}|$ =------

x) If $\vec{a} = \vec{\imath} - \vec{\jmath} + 3\vec{k}, \vec{b} = 2\vec{\jmath} - \vec{k}$, then $\vec{a}.\vec{b}$=------

2. Solve: 5 x 4=20

(i)$(x^2 + y^2)dx + 2xydy = 0$

(ii) $(y^4 + 4x^3y + 3x)dx + (x^4 + 4xy^3 + y + 1)dy = 0$

(iii)$(sinxcosy + e^{2x})dx + (cosxsiny + tany)dy = 0$

(iv) $(1 - x^2)\frac{dy}{dx} - xy = 1$

3. Solve (any four): 5 x 4=20

(i) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$

(ii) $\frac{d^2y}{dx^2} + 4y = x^2$

(iii) $\frac{d^2y}{dx^2} - a^2y = e^{ax}$

(iv) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = x$

(v) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2e^{3x}$

4. a) If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$, find a and b. 6

b) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ show that $A^3 - 23A - 40I_3 = 0$. 7

c) For the matrices $A = \begin{bmatrix} 1 & -4 \\ 0 & 5 \\ 6 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & -7 \end{bmatrix}$ verify that $(AB)^t = B^t A^t$. 7

5. a) Solve the following system of equations by matrix method 7

$$3x - 2y + 3z = 8$$
$$2x + y - z = 1$$
$$4x - 3y + 2z = 4$$

b) Find the inverse of the following matrix by elementary transformations. 7

$$D = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

c) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ by elementary transformations. 6

6. a) Show that the vectors $2\vec{\imath} - \vec{\jmath} + \vec{k}$, $\vec{\imath} - 3\vec{\jmath} - 5\vec{k}$ and $3\vec{\imath} - 4\vec{\jmath} - 4\vec{k}$ form a right angled triangle. 5

b) Find the unit vector in the direction of $2\vec{a} - \vec{b}$ where $\vec{a} = 2\vec{\imath} - 3\vec{\jmath} + 4\vec{k}$ and $\vec{b} = \vec{\imath} - 2\vec{k}$. 5

c) If $\vec{a} = 2\vec{\imath} + \vec{\jmath} + 2\vec{k}$ and $\vec{b} = 5\vec{\imath} + \vec{\jmath} + 2\vec{k}$ then find $\vec{a}.\vec{b}$ and $\vec{a} \times \vec{b}$. 2+3=5

d) If $f(x, y, z) = 3x^2y - y^3z^2$ find $grad.f$ at the point $(1, -2, -1)$. 5

7. a) The position vectors of the points P and Q are $2\vec{\imath} + 3\vec{\jmath} - \vec{k}$ and $4\vec{\imath} - 3\vec{\jmath} + 2\vec{k}$ respectively. Determine $\overrightarrow{PQ}$ and $|\overrightarrow{PQ}|$. 2+2=4

b) Find the angle between the vectors $2\vec{\imath} + 3\vec{\jmath} - \vec{k}$ and $\vec{\imath} - 2\vec{\jmath} - 6\vec{k}$. 5

c) Find the area of a triangle having the points $A(1,1,1), B(1,2,3)$ and $C(2,3,1)$as its vertices. 6

d) If $\vec{F} = x^2y\vec{\imath} + xz\vec{\jmath} + 2yz\vec{k}$ then find $curl\vec{F}$. 5