Total No. of printed pages = 7

BES-2202/EM-II/4th Sem/2013

ENGINEERING MATHEMATICS – II

Full Marks – 100

Pass Marks – 30

Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer any *five* questions.

1. (a) Calculate the co-efficient of correlation between X and Y series from the following data :

X :	80	91	98	71	61	81	70	63
Y :	120	132	151	107	102	131	118	103

Also find two regression equations.

5+5=10

(b) Let A and B be the two possible outcomes of an experiment and supposed

$P(A) = \cdot 6$, $P(A \cup B) = 0 \cdot 9$ and $P(B) = r$.

(i) For what choice of 'r' are A and B mutually exclusive ?

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(ii) For what choice of 'r' are A and B independent ? 3+3=6

(c) If $\sum_{n=2}^{\infty} a_n = a$, then prove that $\sum_{n=1}^{\infty} a_n = a + a_1$.
4

2. (a) If $F(t) = t^2$; $0 < t < 2$ and $F(t + 2) = F(t)$, find $L\{F(t)\}$. 5

(b) Solve the following ordinary differential equation : 6

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$$

(c) Compute Spearman's rank correlation coefficient for the following data : 5

Person	A	B	C	D	E	F	G	H	I	J
Rank in statistics	9	10	6	5	7	2	4	8	1	3
Rank in income	1	2	3	4	5	6	7	8	9	10

(d) The probability that a student will solve a problem is $\frac{3}{4}$ and the probability that another student will solve is $\frac{4}{5}$. What is the probability that neither can solve it ? 4

3. (a) Evaluate any *two* : $2\times4=8$

(i) $L^{-1}\left\{\frac{1}{P^2(P+1)^2}\right\}$

(ii) $L^{-1}\left\{\frac{P+1}{P^2+6p+25}\right\}$

(iii) $L^{-1}\left\{\frac{1}{\sqrt{3p+2}}\right\}$

(b) Test for convergence the series whose n^{th} term is $\left\{(-1)^n\frac{1}{n}\right\}$ 4

(c) Solve : 4+4=8

(i) $\frac{d^2y}{dx^2} + 4y = x^3$

(ii) $(y^2e^{2xy} + 4x^3)\,dx + (2xye^{2xy} - 3y^2)dy = 0$

4. (a) If a series $\sum u_n$ of positive monotonic decreasing terms convergent, then prove that not only $u_n \to 0$ but also $nu_n \to 0$ as $n \to \infty$. 5

(b) There are 8 positive and 9 negative numbers. Six are chosen at random without replacement and multiplied. What is the probability that the product is a negative number ? 7

(c) The students of a college engage in various sports in the following proportions :

Football (F) : 50% of all students

Basketball (B) : 70% of all students

Both (F and B) : 40% of all students.

If a student is selected at random, what is the probability that he will

(i) play football or basketball

(ii) play neither game ? 4+4=8

5. (a) Solve :

(i) $x dy - y dx = \sqrt{x^2 - y^2}\, dx$

(ii) $\dfrac{dy}{dx} + xy = xy^n$ 4+4=8

(b) Evaluate : $L\{\sin\sqrt{t}\}$ 5

(c) (i) Find the radius of convergence of the series : 3

$$x + \frac{x^2}{2^2} + \frac{2!}{3^3}x^3 + \frac{3!}{4^4}x^4 + \ldots\ldots\ldots\ldots$$

(ii) Test the convergence of the following series : 4

$$1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \ldots\ldots\ldots\ldots$$

6. (a) If $L\{F(t)\} = f(s)$, then prove that

$$L\left\{\frac{F(t)}{t}\right\} = \int_p^\infty f(s)\, ds;$$

provided the integral exists. 5

(b) There are 6 white and 9 red balls in a bag. A ball is drawn and then replaced. What is the probability that a white and a red ball are drawn in that order ? 7

(c) "If a series $\sum a_n$ converges to the sum a_0, then so does any series obtained from $\sum a_n$ by grouping the terms in brackets without altering the order of the terms." Prove it. Is converse of the above statement true ? Justify with an example. 4+4=8

7. (a) If $L^{-1}\{f(p)\} = F(t)$, then prove that

$$L^{-1}\{f(ap)\} = \frac{1}{a} F\left(\frac{t}{a}\right)$$

Using the above result evaluate

$$L^{-1}\left\{\frac{32p}{\left(16p^2+1\right)^2}\right\} \text{ if } L^{-1}\left\{\frac{p}{\left(p^2+1\right)^2}\right\} = \frac{1}{2} t \sin t.$$

4+3=7

(b) Solve : 5

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$$

(c) If $\sum \frac{1}{n}$ is divergent, then show that

$$\sum \frac{1}{n^{1+\frac{1}{n}}}$$ 4

(d) Test for convergence the series

$$\sum \frac{n^2-1}{n^2+1} x^n, \; x > 0.$$ 4