

2021

NUMERICAL METHODS FOR ENERGY SYSTEM

Full Marks: 60

Time: 2 hours

The figures in the margin indicate full marks for the questions.

A. Multiple Choice Questions

1 x 10=10

1. If the equation $f(x) = 0$ is such that $g(x) = \phi(x)$ and $\phi'(x_0) = 3$, with the Newton-Raphson method is not convergent for the problem, then which of the following method is usable?
 - a. Iteration Method
 - b. Newton-Raphson Method
 - c. Regula-False Method
 - d. None of these
2. Gauss-Seidel iterative method can be used for solving a set of
 - a. Linear differential equations only
 - b. Linear algebraic equations only
 - c. Both linear and nonlinear algebraic equations
 - d. Both linear and nonlinear algebraic differential equations
3. If $f(x) = 0$ such that $g(x) = \phi(x)$, then iteration Method is convergent if
 - a. $|f'(x)| < 1$
 - b. $|\phi'(x)| < 1$
 - c. $|f''(x)| < 1$
 - d. $|\phi''(x)| < 1$
4. Rate of convergence value of Newton-Raphson method is
 - a. 2.6
 - b. 1.2

- c. 2
 - d. 1
5. For the unequal interval, method use to interpolate is
- a. Newton Forward Interpolation
 - b. Newton Forward Interpolation
 - c. Lagrange interpolation
 - d. Gauss Interpolation
6. Which of the following is not a closed formula for integration?
- a. Trapezoidal Rule
 - b. Simpson's Rule
 - c. Cubic Spline
 - d. None of the above
7. "Quadrature" refers to
- a. A method to obtain roots of a nonlinear equation
 - b. Quadratic approximation of a function
 - c. Inner (dot) product with quadratic weighting functions
 - d. Representing integral as a weighted sum of function values at certain points
8. Which of the following is true about a general ODE Boundary Value Problem (BVP)?
- a. Finite difference approximation leads to linear algebraic equations
 - b. One can always find an analytical solution to any ODE-BVP
Computational Techniques
 - c. Neumann and mixed boundary conditions are handled using "ghost point" approach
 - d. It can be solved using "method of lines"
9. The first and second derivatives of the function at the point $x=1.2$ from the following data
- | | | | | | |
|----|---|---|---|---|---|
| X: | 1 | 2 | 3 | 4 | 5 |
| Y: | 0 | 1 | 5 | 6 | 8 |
- a. 14 and 17
 - b. 15 and 17

c. 17 and 14

d. 17 and 15

10. Value of $\Delta \tan^{-1} x$ is

a. $\tan^{-1} \left\{ \frac{x}{1+hx+x^2} \right\}$

b. $\tan^{-1} \left\{ \frac{h}{1+hx+x^2} \right\}$

c. $\tan^{-1} \left\{ \frac{x}{1+x^2} \right\}$

d. $\tan^{-1} \left\{ \frac{h}{1+x^2} \right\}$

B. Answer any two(02) from the following questions:

4*2=8

1. Explain how you will use Trapezoidal method to numerically evaluate the integral $\int_0^a f(x) dx$?
2. Calculate the value of the integral $\int_4^{5.2} \log_e x dx$ by Simpson's 3/8 th rule taking $h = 0.2$ retaining the numerical values to 7 decimals. Compare the results with exact values.
3. Find the cubic polynomial which takes the following values:

x	0	1	2	3
f(x)	1	2	1	10

C Answer any six(06) from the following questions

7*6=42

1. Solve the non-linear equations, $2x^2 + 3xy + y^2 = 3, 4x^2 + 2xy + y^2 = 30$. Correct to three decimal places, using Newton-Raphson method, given that $x_0 = -3$ and $y_0 = 30$

In the Table below, the values of y are consecutive terms of a series of which 23.6 is the 6th term. Find the first and tenth terms of the series:

x	3	4	5	6	7	8	9
y	4.8	8.4	14.5	23.6	36.2	52.8	73.9

2. Consider the second order value problem $y'' - 2y' + 2y = e^{2t} \sin(t)$ with $y(0) = -0.4$ and $y'(0) = -0.6$. Using the fourth order Runge-Kutta method, find $y(0.2)$

3. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in $0 < x < 5, t \geq 0$,
 given that $u(x, 0) = 20, u(0, t) = 0, u(5, t) = 100$. Compute u for the time-step with $h=1$ by Crank Nicolson method.
4. Show that the following four-point backward difference formula can be used to compute the second derivative:

$$f_2(x_i) = \frac{2f(x_i) - 5f(x_{i-1}) + 4f(x_{i-2}) - f(x_{i-3})}{h^2}$$

5. Use Romberg's method to compute $I = \int_0^1 \frac{1}{1+x} dx$ correct to three decimal places with Trapezoidal rule.
6. Evaluate the integral $\int_0^{\pi/2} \sin x dx$ using two-terms Gaussian formula
7. Find $f'(5)$ from the following table:

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	4				
2	26	11			
3	58	32	7		
4	112	54	11	1	0
7	466	118	16	1	0
9	992	228	22	1	