

2021

MATHEMATICS-II

Full Marks: 60

Time: Two hours

The figures in the margin indicate full marks for the questions.

A. Multiple Choice Questions

1 x 20=20

1. If $f(x) = 2x^2 + 3x - 4$, then the value of $\frac{d}{dx}f(2)$ is
 - a 11
 - b 12
 - c 10
 - d None of the above
2. Value of k for which $\lim_{x \rightarrow 2} f(x)$ exist, where $f(x) = \begin{cases} x^2 + 1, & x \leq 2 \\ x + k, & x > 2 \end{cases}$ is
 - a. $k = -3$
 - b. $k = 2$
 - c. $k = 3$
 - d. $k = 0$
3. The value of $\frac{d}{dx}|x|$ is equal to
 - a. $\frac{x}{|x|}, x \neq 0$
 - b. 1
 - c. -1
 - d. None of these
4. $\int \frac{4x-3}{2x^2-3x+4} dx$ is equal to
 - a. $\log(2x^2 - 3x + 4) + C$

- b. $\log(4x - 3) + C$
- c. $2 \log(2x^2 - 3x + 4) + C$
- d. $4 \log(2x^2 - 3x + 4) + C$
5. The middle point of the line joining the points (2, -3) and (-4, 5) is
- (1, 1)
 - (1, -1)
 - (-1, 1)
 - (1, 2)
6. The distance between the points (2, 4) and (6, k) is 5 then k equals
- 6 or 1
 - 7 or 1
 - 8 or 2
 - 10 or 2
7. The equation of the line making an intercept 3 on the Y-axis and an angle of 30° with the positive direction of X-axis is
- $y = \frac{\sqrt{3}}{2}x + 3$
 - $y = \frac{1}{\sqrt{2}}x + 3$
 - $y = \frac{1}{\sqrt{3}}x + 3$
 - $y = \frac{1}{3}x + 3$
8. The gradient of the line passing through the points (-1, 2) and (2, -4) is
- 3
 - 1/3
 - 0
 - 3
9. The point of intersection of the lines $x + y - 1 = 0$ and $2x + 3y - 5 = 0$ is
- (-2, 3)

- b. $(-2, -3)$
- c. $(2, -3)$
- d. $(2, 3)$
10. The angle between the lines $5x - 4y + 7 = 0$ and $4x + 5y - 6 = 0$ is
- a. 30°
- b. 45°
- c. 60°
- d. 90°
11. The circle $x^2 + y^2 - 6x - 8y - 11 = 0$ has centre and radius as
- a. $(3, -4); 6$
- b. $(-3, 4); 5$
- c. $(3, 4); 6$
- d. $(-3, -4); 5$
12. Consider the function, $f(x) = \begin{cases} -x^2, & x \leq 0 \\ x^2, & x > 0 \end{cases}$, then
- a. Continuous at $x = 0$ but not differentiable at $x = 0$
- b. Continuous and differentiable at $x = 0$
- c. Neither continuous nor differentiable at $x = 0$
- d. Differentiable but not continuous at $x = 0$
13. The equation of the tangent to the circle $x^2 + y^2 + 6x - 2y - 7 = 0$ at the point $(-4, -5)$ is
- a. $x + 6y - 15 = 0$
- b. $x - 6y - 14 = 0$
- c. $x + 6y + 15 = 0$
- d. $x + 6y + 14 = 0$
14. The equation of the parabola with the focus $(2, 0)$ and directrix $x = 0$ is
- a. $y^2 - 4x + 4 = 0$

b. $y^2 + 4x + 4 = 0$

c. $y^2 - 4x - 4 = 0$

d. $y^2 + 4x - 4 = 0$

15. The latus rectum of the ellipse $2x^2 + 3y^2 = 1$ is

a. $\frac{\sqrt{2}}{3}$

b. $\frac{2\sqrt{2}}{3}$

c. $\frac{2\sqrt{2}}{\sqrt{3}}$

d. $\frac{2}{\sqrt{3}}$

16. Value of $\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-1}{x}$ is

a. 0

b. 1

c. $\frac{1}{2}$

d. None of these

17. Consider the function, $f(x) = \begin{cases} -2x - 1, & \text{when } x < -3 \\ 5, & \text{when } -3 \leq x < 2 \\ 2x + 1, & \text{when } x \geq 2 \end{cases}$, then

a. $f(x)$ is differentiable at $x = -3$

b. $f(x)$ is continuous at $x = -3$ but not differentiable at $x = -3$

c. Neither continuous nor differentiable at $x = -3$

d. None of these

18. $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ is equal to

a. $\sec(xe^x) + C$

b. $\tan(xe^x) + C$

c. $\sin(xe^x) + C$

d. $\cot(xe^x) + C$

19. The domain of the function of the function $f(x) = \frac{2}{(x-2)(x-3)}$ is

- a. $\mathbb{R} - \{2, 3\}$
- b. $\{2, 3\}$
- c. \mathbb{R} , The set of real numbers
- d. None of these

20 The function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(x) = x^2 + 1$ is

- a. neither one-one nor onto
- b. onto but not one-one
- c. Bijective
- d. one-one but not onto

B. Very Short Question

2*6=12

1. Evaluate

$$\int_0^{\pi/2} \tan x dx$$

2. Evaluate $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$

3. Find $\frac{d^2y}{dx^2}$ if $y = \sec(\tan^{-1}x)$

OR

Evaluate $\frac{dy}{dx}$ if $x^y = y^x$

4. Find the coordinates of the point which divides internally the line joining the points $(-1, 2)$ and $(2, 3)$ in the ratio 1:2.

5. Find the gradient and equation of the line joining the points $(-5, 2)$ and $(7, 0)$.

6. Find the equation of the circle with centre $(-1, 2)$ and radius 3 units.

OR

Find the equation of the parabola with focus $(-1, 2)$ and directrix $x - 2y + 3 = 0$.

C Short Question

4*7=28

1. Test the continuity of the following function

$$f(x) = \begin{cases} x+1 & ;x < 2 \\ 2 & ;x = 2 \\ 3-x & ;x > 2 \end{cases} \quad \text{at the point } x = 2$$

- 2 Using definition, find the derivatives of the following functions (any one)

(i) $f(x) = \log x$

(ii) $f(x) = \sin x$

- 3 Evaluate the following limit

$$\lim_{x \rightarrow \infty} \frac{x}{\sqrt{4x^2 + 1} - 1}$$

4. Integrate (any one)

(i) $\int \frac{1}{x^2+x+1} dx$

(ii) $\int \sin^4 x dx$

(iii) $\int x^2 \sin x dx$

5. Show that the points $(2, 1)$, $(-2, 4)$, $(-5, 0)$ and $(-1, -3)$ are the vertices of a square.

6. Find the equation of the straight line passing through the point of intersection of the straight lines $2x - y - 3 = 0$ and $x - 2y - 4 = 0$ and parallel to the straight line $4x + y + 5 = 0$.

- or Find the equation of the circle through the points $(1, 1)$, $(2, -1)$ and $(3, 2)$

7. Find the major axis, minor axis, distance between the foci, eccentricity and length of the latus rectum of the ellipse $2x^2 + 5y^2 = 10$.