Total number of printed pages–7

53 (EC 302) LSYS

2019

LINEAR SYSTEMS AND SIGNALS

Paper : EC 302

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* A triangular pulse signal $x(t)$ is depicted in *Fig.* (1).

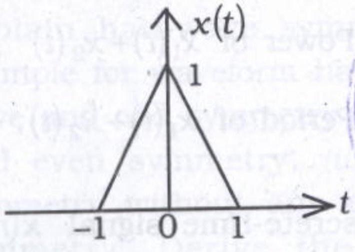

Fig. (1)

Evaluate and plot $x(3t+2)$ and $x(-2t-1)$. 6

Contd.

(b) Show that derivative of unit step function is a unit impulse function. Plot the function $x(t)=u(t)-2u(t-1)+u(t-2)$ and evaluate its first derivative. 6

(c) Under what condition can we say a set of functions form an orthogonal set? Given a real valued function $f(t)$, discuss how one can approximate it as a linear combination of functions in an orthogonal set. 8

2. (a) Given $x_1(t)=\sin 3\pi t$, $x_2(t)=\sin 2\pi t$.

Find :

(i) Power of $x_1(t)+x_2(t)$

(ii) Period of $x_1(t)+x_2(t)$. 6

(b) A discrete-time signal $x(n)=c.r^n$. If $r=2$ and $c=4$, find the even and odd parts of $x(n)$, and plot them as a function of n. 6

(c) Check for linearity, time-invariance and causality of a system represented by the following input-output relationship.

$$y(t)=|x(t+\tau)|$$

Under what condition the above system requires no memory? 8

3. (a) Show that the zero-state output of an LTI system with impulse response $h(t)$ and input $x(t)$ is given by the convolution integral $y(t)=x(t)*h(t)$.

Find $y(t)$ if $h(t)=e^{-t^2}$ and $x(t)=3t^2$ for all t. 10

(b) Explain half-wave symmetry. Give example for waveform having (i) half-wave and odd symmetry (ii) half-wave and even symmetry, (iii) half-wave symmetry without any even or odd symmetry. Derive the simplified expressions for non-zero trigonometric Fourier series coefficients in each of the above case. 10

4. (a) If a periodic signal $x(t)$ is expressed in terms of exponential Fourier series with the coefficients denoted as X_n, where n is any integer. Find the power of the signal $x(t)$ in terms of X_n. If this signal is given as input to an LTI system with frequency response $H(w)$ evaluate the output signal. 10

(b) Write the analysis and synthesis equations for Discrete-time Fourier series. Determine DTFS coefficients of the signal $x(n)$ and plot its magnitude and phase spectrum.

$$x(n) = 1 + \sin\left(\frac{2\pi}{7}n\right) + 3\cos\left(\frac{2\pi}{7}n\right) + \cos\left(\frac{4\pi}{7}n + \frac{\pi}{2}\right).$$

10

5. (a) Find the inverse Fourier transform of $X(\omega) = \delta(\omega - \omega_0)$ and using this result evaluate the Fourier transform of 1. 6

(b) Consider the triangular pulse signal depicted in *Fig.* (2).

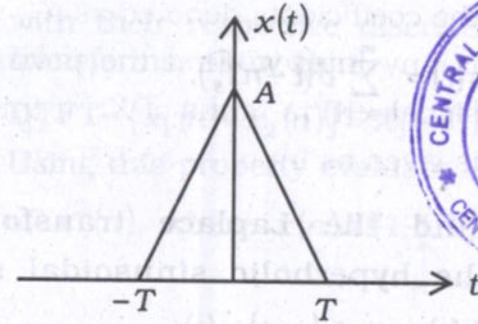

Fig. (2)

Find the Fourier transform of $x(t)$ and plot its magnitude spectrum. 6

(c) If $x(t) \rightleftharpoons X(\omega)$ are Fourier transform pairs, find the Fourier transform of

(i) $x(at)$

(ii) $tx(t)$. 8

6. (a) Describe the time-domain and frequency domain characteristics of an ideal low pass filter. What is Paley-Wiener Criterion ? Explain its implications taking the example of ideal low pass filter. 8

(b) Find the Laplace transform of an impulse train signal

$$s(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT_s).$$ 6

(c) Find the Laplace transform of the hyperbolic sinusoidal signal, $x(t)=\sinh(\omega_0 t)\cdot u(t)$. 6

7. (a) Determine the z-transform of

$$x(n)=-\left(\frac{1}{2}\right)^n u(-n-1)+2^n u(-n-1)$$

and depict the ROC. 6

(b) Determine the inverse z-transform of $X(z)=\dfrac{1}{1-1\cdot5z^{-1}+0\cdot5z^{-2}}$

if ROC : $0\cdot5<|z|<1$. 6

(c) If $x_1(n)\longleftrightarrow X_1(e^{j\omega})$ and $x_2(n)\longleftrightarrow X_2(e^{j\omega})$ are two signals with their respective discrete Fourier transforms, show that

$$\text{DTFT }\{x_1(n)*x_2(n)\}=X_1(e^{j\omega})\cdot X_2(e^{j\omega})$$

Using this property evaluate the DTFT of $x^2(n)$, where $x(n)=\{1,\underset{\uparrow}{1},1\}$. 8
