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53 (MA 101) ENMA

2019

ENGINEERING MATHEMATICS - I

Paper : MA 101

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer **any five** questions.

1. (a) State limit comparison test. Examine the convergency of the following series.

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt[3]{n+2}} \quad 2+4=6$$

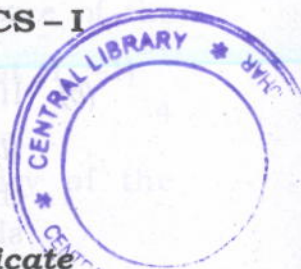
- (b) Find the equation of the sphere which passes through the points (1, 2, 3), (0, -2, 4), (4, -4, 2) and (3, 1, 4).

7

- (c) Show that, every absolutely convergent series is convergent. Give an example with justification to show that the converse is not true in general.

5+2=7

Contd.



2. (a) Find the equation of the plane which contains the line $\frac{x-3}{1} = \frac{y-1}{-1} = \frac{z}{3}$ and is perpendicular to the plane $x+y+2z=4$. 5

(b) If (l_1, m_1, n_1) , (l_2, m_2, n_2) and (l_3, m_3, n_3) be the direction cosines of three mutually perpendicular lines, show that the line whose direction ratios are $l_1+l_2+l_3$, $m_1+m_2+m_3$ and $n_1+n_2+n_3$ makes equal angles with them. 6

(c) Test the following series:

$$\sum_n \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$$

4

(d) Find the equation of the plane through the line of intersection of the planes $x+2y-3z=1$, $2x-3y+2=0$ and the origin. 5

3. (a) Expand $\log_e x$ in powers of $(x-1)$ and hence evaluate $\log_e(1.1)$ correct to four decimal places. 5+1=6

(b) Find all the asymptotes of the curve

$$y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 - 1 = 0$$

6

(c) Find the centre of curvature of the curve $y = x^3 - 6x^2 + 3x + 1$ at $(1, -1)$ 4

(d) Investigate the continuity of the following function:

$$f(x, y) = \begin{cases} \frac{x^2 - y^2}{x^2 + y^2} & ; \text{ if } (x, y) \neq (0, 0) \\ 0 & ; \text{ if } (x, y) = (0, 0) \end{cases}$$

at the origin. 4

4. (a) Solve the following differential equations (any two): $4 \times 2 = 8$

(i) $(a+x) \frac{dy}{dx} = y - ay^2$

(ii) $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$

(iii) $\left\{ y \left(1 + \frac{1}{x}\right) + \cos y \right\} dx + (x + \log x - x \sin y) dy = 0$



6. (a) Solve: (any two) 4×2=8

(i) $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = x$

(ii) $(D^2 + 6D + 9)y = 2e^{-3t}$, $D = \frac{d}{dt}$

(iii) $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = \cos 3x$

Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the major axis. 6

(c) Solve the following simultaneous differential equation:

$\frac{dx}{dt} + 2x - 3y = 0$

$\frac{dy}{dt} - 3x + 2y = 0$ 6



(b) If $y = e^{a \sin^{-1} x}$, show that

$(-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0$ 5

(c) Verify that $x dx + y dy = \frac{a^2 (x dy - y dx)}{x^2 + y^2}$ is exact and hence solve it. 5

(d) Apply Leibnitz theorem to $y = e^{ax} \cdot x^5$ to find y_5 . 2

5. (a) Find the radius of curvature of $\sqrt{x} = \sqrt{r} \cos \frac{\theta}{2}$ at (r, θ) . 6

(b) Show that $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} xy \frac{x^2 - y^2}{x^2 + y^2} = 0$ 5

(c) Show that the length of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ is $6a$. 7

(d) Evaluate: $\int_0^{2a} x^{5/2} \sqrt{2a-x} dx$ 2

(a) Solve (any two) $xy' + y = 2x$ and $x^2y' - xy = x^2 - 8$

$$(i) \quad xy' + y = 2x$$
$$(ii) \quad x^2y' - xy = x^2 - 8$$

$$(i) \quad D_x + D_y + y = 2x \quad D = \frac{d}{dt}$$

If we let $u = xy$ then

$$(ii) \quad \frac{d}{dx} \left(\frac{y}{x} \right) = \frac{x^2 - 8}{x^2}$$

Let

Find the volume of the solid generated

by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

about the major axis.

(c) Solve the following simultaneous differential equations

$$\frac{dx}{dt} + 2x - 3y = 0$$

$$x + y + z = 1$$

$$\frac{dy}{dt} - 3x + 2y = 0$$

$$x^2 + y^2 + z^2 = 1$$