END SEMESTER EXAMINATION, NOVEMBER-2018

Semester : 3rd

Subject Code : Sc 303

MATHEMATICS – III

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks
for the questions.

Instructions :

1. *All* the questions of PART – A are compulsory.

2. Answer any *five* questions from PART – B.

PART – A

Marks – 25

1. Fill in the blanks :   1×10=10

    (a) Order of the equation
    $$\frac{d^4y}{dx^4} + 4x\left(\frac{dy}{dx}\right)^2 - 6y = 3$$
    is ——.

(b) Order of the equation $\dfrac{d^2y}{dx^2} + 4\left(\dfrac{dy}{dx}\right)^4 - y = 2$

is ——.

(c) Degree of the equation

$$\dfrac{d^2y}{dx^2} + xy\left(\dfrac{dy}{dx}\right)^4 + 2y = 2 \quad \text{is} \quad ——.$$

(d) Solution of $(\sec x)\dfrac{dy}{dx} + y = 0$ is ——.

(e) Solution of $e^x\dfrac{dy}{dx} + y = 0$ is ——.

(f) Solution of $x^2 y^2 \dfrac{dy}{dx} + 4 = 0$ is ——.

(g) Solution of $\sec^2 x \tan y\, dx + \sec^2 y \tan x$

$dy = 0$ is ——.

(h) Primitive of $x\,dx + y\,dy = 0$ is ——.

(i) Integrating factor of $\dfrac{dy}{dx} + \sec x \cdot y = \tan x$

is ——.

(j) Integrating factor of $\dfrac{dy}{dx} + \dfrac{y}{x} = x$ is ……..

2. Write true or false : $1 \times 10 = 10$

(a) $(x + y^2)\, dx + (2xy - e^y)\, dy = 0$ is an exact equation.

(b) Auxiliary equation of $\dfrac{d^2y}{dx^2} + 9y = x^2$ is $m^2 + 9x = 0$.

(c) Solution of $y = px + p^2$ is $y = cx + c^2$.

(d) Formula for mode is $1 + \dfrac{f - f_1}{2f - f_{-1} - f_2}$

(e) Formula for median is

$$\sqrt{\dfrac{1}{N}\sum fd^2 - \left(\dfrac{1}{N}\sum fd\right)^2}$$

(f) Quartile deviation is a measure of central tendency.

(g) Mean of 4, 6, 1, 11, 3, 5, 8, 2 is 8.

(h) When tossing a coin getting a head and getting a tail are mutually exclusive events.

(i) Probability of drawing a red ball from a box containing 8 black balls and 2 red balls is $\frac{1}{4}$.

(j) Probability of drawing a Red Queen from a pack of cards is $\frac{4}{52}$.

3. Choose the correct answer : $1 \times 5 = 5$

(a) Transpose of $\begin{pmatrix} 1 & 0 & -2 \\ 3 & -1 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 2 \\ 1 & 3 & -5 \end{pmatrix}$ is

(i) $\begin{pmatrix} 1 & 4 \\ 0 & -1 \end{pmatrix}$

(ii) $\begin{pmatrix} 3 & 1 \\ 0 & -2 \\ 1 \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & -2 \\ -1 & 2 \\ 0 & 1 \end{pmatrix}$

(iv) $\begin{pmatrix} 1 & 0 & 2 \\ 1 & 4 & 1 \end{pmatrix}$

(b) 3rd order identity matrix is

(i) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$

(ii) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

(iv) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(c) Characteristic equation of $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ is

(i) $\lambda^3 + 3\lambda - 1 = 0$

(ii) $(\lambda - 1)^2 (\lambda - 5) = 0$

(iii) $\lambda^3 + 3\lambda^2 + \lambda 5 - 1 = 0$

(iv) $\lambda^2 + 2\lambda + 7 = 0$

(d) (4, 0) is a solution of

(i) $2x - y < 1$

(ii) $x + y \geq 6$

(iii) $x + y \geq 4$

(iv) $x + y > 4$

(e) $(1, 6)$ is a solution of

(i) $x - y < 1$ (ii) $2x - 3y \geq 6$

(iii) $x + y \geq 8$ (iv) $3x + y > 9$

PART – B

Marks – 45

4. (a) Find order and degree of the following differential equations : $2+2=4$

(i) $\dfrac{d^2y}{dx^2} - 2\left(\dfrac{dy}{dx}\right)^2 + y = 0$

(ii) $x\left(\dfrac{d^3y}{dx^3}\right)^3 - x^2\left(\dfrac{dy}{dx}\right)^4 + y^6 = 0$

(b) Given $f(x, y) = 5x^4 + 3x^3y - e^{xy} \sin y$.

Find

(i) $\dfrac{\partial f}{\partial x}$ (ii) $\dfrac{\partial f}{\partial y}$ $2+3=5$

5. Solve the following : $3 \times 3 = 9$

(a) $\dfrac{dx}{x} = \sin y \cdot dy$

(b) $(e^y + 3) \sin x \, dx + e^y \cos x \, dy = 0$

(c) $(2x + y + 3) \, dy = (2x + y - 3) \, dx$

6. Solve the following : $3 \times 3 = 9$

(a) $\dfrac{dy}{x} + 2xy = 2e^{-x^2}$

(b) $\dfrac{dy}{dx} + \tan x \, \tan y = \cos x \, \sec y$

(c) $(2x - y) \, dx = (x - y) dy$

7. Solve the following : $3 \times 3 = 9$

(a) $\dfrac{d^3y}{dx^3} + y = 0$

(b) $\dfrac{d^2y}{dx^2} - y = 0$, if $y = 0$, $\dfrac{dy}{dx} = 0$ at $x = 0$

(c) $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x}$

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8. (a) Find median and mean deviation from median of the following data : 3+3=6

Height in cm :	80-90	90-100	100-110	110-120	120-130	130-140	140-150
No. of Students:	25	28	38	50	31	23	5

(b) Find probability that a leap year will have 53 Sundays. 3

9. (a) Find mean of :

23, 56, 125, 67, 87, 46, 99, 54, 31, 19. 3

(b) Find standard deviation :

60, 80, 75, 72, 69, 74, 65, 62. 3

(c) Three machines A,B,C manufacture respectively 0.6, 0.3 and 0.1 of the total production.

The percentage of defective items produced by A, B, C is 3, 2, 1 per cent respectively. For an item chosen at random what is the probability that it is defective ? 3

10. (a) Draw graph :

(i) $y = \sin 4x$ $-2\pi \leq x \leq 4\pi$ 4

(ii) $y = \tan x$ $\dfrac{-\pi}{4} \leq x \leq \dfrac{\pi}{4}$ 3

11. (a) Find transpose of $\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$ 2

(b) Find inverse of $\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$ 2+2=4

(i) $\begin{pmatrix} a & -c & 1 \\ b & b & 0 \\ -a & -b & -1 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & -\omega & 1 \\ -1 & \omega^2 & 0 \\ \omega^4 & \omega & -1 \end{pmatrix}$

(b) Solve using matrix method :

$3x - y + 2z = -2$
$x + 2y + z = 1$
$5x + 3y + 2z = 4$ 5

12. (a) Solve : $x - 5y \leq -1$ 4

(b) Maximize $z = 3x + 2y$

Subject to $x + 2y \leq 10$,
$3x + y \leq 15$,
$x \geq 0, y \geq 0$. 5