

2018

DIGITAL SIGNAL PROCESSING

Paper : EC 603

Full Marks : 100

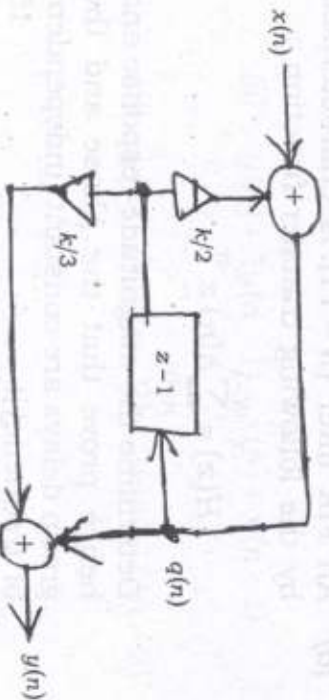
Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer **any five** questions.

1. (a) Establish the relationship between analog frequency and digital frequency. 5

- (b) Consider the discrete-time system shown below. For what values of 'K' is the system BIBO stable ? 15



Contd.

2. Determine the output sequence of the system with impulse response $h(n) = \left(\frac{1}{2}\right)^n u(n)$, when the input is the complex exponential sequence $x(n) = 4 \exp(j\pi n/2)$; $-\infty < n < \infty$. Deduce the necessary theory. 15+5

3. (a) If a discrete-time LTI system is BIBO stable, show that the ROC of its system function $H(z)$ must contain the unit circle, i.e. $|z| = 1$. 10

- (b) Find the circular convolution of the two sequences : $x(n) = \{1, -2, 4, 1, 5\}$ and $h(n) = \{3, 0, -2, 5\}$ using graphical method. 10

4. (a) An FIR filter ($N = 11$) is characterised by the following transfer function :

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

Determine the magnitude response and hence prove that the phase and the group delays are constant (independent of frequency). 15

- (b) Design a single-pole low-pass digital filter with a 3dB bandwidth of 0.2π using Bilinear transformation. The corresponding analog filter has a system response given by $H(s) = \Omega_c / (s + \Omega_c)$; where ' Ω_c ' is the 3dB bandwidth of the analog filter. 5

5. Find the impulse response $h(n)$ for each of the causal, discrete-time LTI systems satisfying the following difference equations and also indicate the type of the system (FIR or IIR) : 20

(i) $y(n) = x(n) - 2x(n-2) + x(n-3)$

(ii) $y(n) + 2y(n-1) = x(n) + x(n-1)$

(iii) $y(n) - 0.5y(n-2) = 2x(n) - x(n-2)$

6. Show that analog transfer function

$$Ha(s) = \frac{b \cdot s}{s^2 + bs + \Omega_0^2}; \quad b > 0$$

has a band-pass magnitude response with $|Ha(j0)| = |Ha(j\infty)| = 0$ and $|Ha(j\Omega_0)|$ is unity.

Also determine the frequencies Ω_1 and Ω_2 at which the gain is 3dB below the maximum value of 0dB at Ω_0 . Show that $\Omega_1, \Omega_2 = \Omega_0^2$.

Hence show that $b = \Omega_2 - \Omega_1$; which is the 3dB bandwidth of the bandpass transfer function.

10+4+3+3
