Total number of printed pages–6

53 (EC 302) SISY

2018

LINEAR SYSTEMS AND SIGNALS

Paper : EC 302

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions out of* **seven.**

1. (a) Discuss how to represent electromagnetic wave as a signal. 4

   (b) Differentiate between the following classification of signals —

   (i) continuous time and discrete time

   (ii) analog and digital signals. 6

Contd.

(c) Find the even and odd part of the signals : 5

(i) $x_1(t) = e^{-at}u(t)$

(ii) $x_2(t) = e^{-at}$

Given 'a' is a positive real number.

(d) If $x(t)$ is given as in *Fig.* (1), evaluate and plot the signal $-x\left(-2t+\frac{3}{2}\right)$. 5

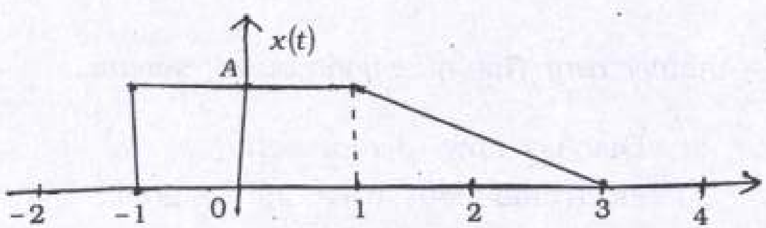

Fig. (1)

2\. (a) Define impulse function. Discuss how this function becomes important in the study of Linear Time Invariant (LTI) systems. 5

(b) Construct the signal given in *Fig.* (1) in terms of shifted and scaled unit step functions. 5

(c) Check the linearity, time-invariance and causality of the systems represented by the following input-output relations.

(i) $y(t) = \int_{-\infty}^{t} x(t) \cdot dt$

(ii) $y(t) = x(2t) + 4x(t)$

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3\. (a) What do you understand by an orthogonal set of complex functions? Derive the value of coefficients which will linearly approximate a given function $f(t)$ in terms of a set of N-orthogonal functions ; $\{g_1(t), g_2(t), \ldots, g_N(t)\}$ as $\sum_{i=1}^{N_1} c_i g_i(t)$.

10

(b) Expand the periodic half-wave rectifier output waveform in terms of exponential Fourier series. Assume the time period, $T = 2\pi$ and the amplitude, A. 10

4. *(a)* Evaluate the Fourier transform of a triangular pulse function and plot its spectrum. 6

(b) Show that convolution of two signals in time domain is equivalent to multiplication of their Fourier transforms. 4

(c) Evaluate the Fourier transform of signum function as a limiting case of exponential decaying functions. Use this result then to evaluate the Fourier transform of unit step function. 7

(d) Discuss the conditions under which the Fourier transform of a function does not exist. 3

5. *(a)* State and prove Parseval's theorem for energy signals. 5

(b) Explain Paley-Weiner criterion with the help of an example. 5

(c) Explain how to evaluate the Fourier transform of a periodic signal with the help of an impulse train signal of period *T*. 5

(d) State and prove sampling theorem. 5

6. *(a)* Find the unilateral Laplace transform of $\frac{d^n x(t)}{dt^n}$. 4

(b) Evaluate the zero-state and zero-input response of a system represented by the following differential equation.

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6 = u(t)$$

Given $x(0^-) = 0;\quad x'(0^-) = -2.$ 6

(c) Evaluate the convolution of the following signals x_1 and x_2.

(i) $x_1(t) = e^{-at}\,u(t)$

and $x_2(t) = e^{-bt}\,u(t)$

(ii) $x_1(n) = \{1, \underset{\uparrow}{0}, -1\}$

and $x_2(n) = \{\underset{\uparrow}{2}, 2, 3\}$. 6

(d) State and prove final value theorem. 4

7. (a) Find the Z-transform of $x(n) = n2^n\ u(n)$ and specify its region of convergence. 5

(b) Find the inverse Z-transform of $X(z) = \dfrac{z(2z+1)}{z^2+5z+6}$ if it correspond to a causal signal. 5

(c) Find the Discrete-Time Fourier Series (DTFS) expansion of the signal

$$x(n) = 4\sin(3n) + \frac{3}{2}\cos(4n).$$

5

(d) Starting with DTFS synthesis and analysis equations, derive the similar equations for Discrete-time Fourier Transform. 5