Total number of printed pages–7

53 (MA 201) ENMA-II

2018

ENGINEERING MATHEMATICS–II

Paper : MA 201

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* If $u = \dfrac{x}{y-z}$, $v = \dfrac{y}{z-x}$ and $w = \dfrac{z}{x-y}$, then show that $\dfrac{\partial(u,v,w)}{\partial(x,y,z)} = 0$. 4

(b) Show that, $\beta(l,m) = \dfrac{\Gamma l \, \Gamma m}{\Gamma (l+m)}$. 6

Contd.

(c) Find the Fourier series of the function $f(x) = |x|$ in the interval $-2 < x < 2$. 7

(d) Evaluate $\int_0^\infty x^{1/4} . e^{-\sqrt{x}} dx$. 3

2\. (a) A problem of statistics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved ? 4

(b) Find the value of x, y, z, s and t if

$$A = \begin{pmatrix} x & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & y \\ z & s & t \end{pmatrix}$$ is orthogonal. 5

(c) Prove that,

$$grad\ div\left(r^n \bar{r}\right) = n(n+3)r^{n-2} . \bar{r} .$$ 5

(d) Obtain the half-range Fourier sine series for the function $f(x) = x^2$ in the interval $0 < x < 3$. 6

3\. (a) Reduce the following matrix to its normal form

$$\begin{pmatrix} 2 & 3 & -2 & 4 \\ 3 & -2 & 1 & 2 \\ 3 & 2 & 3 & 4 \\ -2 & 4 & 0 & 5 \end{pmatrix}_{4\times 4}$$ 7

(b) Show that a real 2×2 normal matrix is either symmetric or the sum of a scalar matrix and a skew-symmetric matrix. 5

(c) Find a unit normal vector to the level surface $x^2y + 2xz = 4$ at the point (2,–2,3). 3

(d) Evaluate

$$\iint_S x^2 dy\, dz + y^2 dz\, dx + 2z(xy - x - y) dx\, dz,$$

where S is the surface of the cube $0 \le x \le 1$, $0 \le y \le 1$ and $0 \le z \le 1$. 5

4. (a) Find the inverse of the following matrix by elementary row transformation

$$\begin{pmatrix} 1 & -1 & 2 \\ 1 & 2 & 3 \\ 3 & -4 & -5 \end{pmatrix}_{3\times 3}$$ 6

(b) Show that the diagonal elements of a Hermitian matrix are real. 2

(c) Show that the function

$$f(x) = \begin{cases} 0 & \text{for } x < 2 \\ \frac{1}{18}(2x+3) & \text{for } 2 \le x \le 4 \\ 0 & \text{for } x > 4 \end{cases}$$

is a probability density function. Also find its mean. 3+3=6

(b) If $\vec{F} = (2x+y)\hat{i} + (3y-x)\hat{j}$, evaluate $\int_C \vec{F}.d\vec{r}$, where C is the curve in the xy-plane consisting of the straight lines from (0,0) to (2,0) and then to (3,2). 6

5. (a) Reduce the matrix to its row echelon form and hence find the rank

$$\begin{pmatrix} 2 & 4 & 3 & -2 \\ -3 & -2 & -1 & 4 \\ 6 & -1 & 7 & 2 \end{pmatrix}_{3\times 4}$$ 7

(b) Evaluate by Stokes' theorem : $\oint_C (yz\,dx + xz\,dy + xy\,dz)$, where C is the curve $x^2 + y^2 = 1$, $z = y^2$. 3

(c) Find the moment generating function of $f(x) = \frac{1}{a}e^{-x/a}$; $0 \le x \le \infty$, $C > 0$. Also find its variance. 3+4=7

(d) If $|\vec{r}| = r$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that $\vec{\nabla}\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$. 3

6. (a) Find the rank of the following matrix

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{pmatrix}_{3\times 4}$$ 5

(b) If $\vec{a}$ is a differentiable vector function of the scalar variable t and if $|\vec{a}| = a$, then prove that $\vec{a}.\frac{d\vec{a}}{dt} = a\frac{da}{dt}$. 2

(c) Show that the vector $\vec{V} = (\sin y + z)\hat{i} + (x\cos y - z)\hat{j} + (x - y)\hat{k}$ is irrotational. 2

(d) Find median and mode from the following data :

Class Interval :	0–5	5–10	10–15	15–20	20–25
Frequency :	2	4	9	8	3

3+3=6

(e) If $\hat{a}$ and $\hat{b}$ are unit vectors, w is a constant and $\vec{r}$ is a vector function of scalar variable t given by

$\vec{r} = \cos wt\,\hat{a} + \sin wt\,\hat{b}$, show that

$$\frac{d^2\vec{r}}{dt^2} + w^2\vec{r} = \vec{0}.$$ 2

(f) For a binomial distribution, find $E(X)$. 3
