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53 (MA 101) ENMA-I

2018

ENGINEERING MATHEMATICS-I

Paper : MA 101

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* State D'Alemberts Ratio test. Examine the convergency of the series

$$\sum_n \frac{1.2.3....n}{7.10.13....(3n+4)}.$$

2+4=6

(b) Find the equation of the plane through the line $\frac{x-1}{3}=\frac{y+2}{1}=\frac{z-3}{-4}$ and parallel to the line $\frac{x+3}{4}=\frac{y-2}{-2}=\frac{z+1}{5}$.

7

Contd.

(c) Show that the lines whose direction cosines are given by the equations $l+m+n=0$ and $al^2+bm^2+cn^2=0$ are parallel if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$. 7

2. (a) If $y=a\cos(\log x)+b\sin(\log x)$, show that

$$x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0.$$ 7

(b) Expand $\sin x$ in power of $\left(x-\frac{\pi}{2}\right)$ and determine $\sin 91^\circ$, correct to four decimal places. 6

(c) Find the asymptotes of the curve

$$x^3+3x^2y-4y^3-x+y+3=0.$$ 7

3. (a) Form the differential equation of the following equations 4×2=8

(i) $Ax^2+By^2=1$, A, B are arbitrary constants

(ii) $y=e^x(A\cos x+B\sin x)$, where A, B are arbitrary constants.

(b) Solve : ***(any two)*** 6×2=12

(i) $(x+1)\frac{dy}{dx}=x(y^2+1)$

(ii) $e^x\tan y\,dx+(1-e^x)\sec^2 y\,dy=0$

(iii) $3\frac{dy}{dx}+\frac{3y}{x}=2x^4y^4$

4. (a) Is the series $\frac{6}{1.3.5}+\frac{8}{3.5.7}+\frac{10}{5.7.9}+\ldots$ convergent ? Justify. 5

(b) Find the equation of the sphere through the points (0,0,0), (0,1,–1), (–1,2,0) and (1,2,3). 5

(c) Solve the simultaneous differential equation $\frac{dx}{dt}+\frac{dy}{dt}+y=1$ and

$$\frac{dx}{dt}+\frac{dy}{dt}+x=0.$$ 5

(d) Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=\sin h(x)$. 5

5. (a) Find the radius of curvature at any point θ of the curve $x=a(\theta-\sin\theta)$ and $y=a(1-\cos\theta)$. 5

(b) Show that the function

$u=xy+\frac{a^3}{x}+\frac{a^3}{y}$ has a minimum value

at (a,a). 5

(c) Show that the series $\sum_n (-1)^{n-1}\sin\left(\frac{1}{n^2}\right)$ is absolutely convergent. 5

(d) Find the equation of the plane which passes through the point (1,2,3) and is perpendicular to the planes $x-y+3z=12$ and $3x+y-z=0$. 5

6. (a) Find the order and degree of the differential equation — 2×2=4

(i) $\cos x\frac{d^2y}{dx^2}+\sin x\left(\frac{dy}{dx}\right)+y=\tan x$

(ii) $\frac{d^2y}{dx^2}+\exp\left(\frac{dy}{dx}\right)+y=\log x$

(b) Solve : 3×2=6

(i) $\frac{d^2y}{dx^2}-\frac{dy}{dx}-6y=e^{3x}$

(ii) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x$

(c) Find the centre of curvature of the curve $y = x^3 - 6x^2 + 3x + 1$ at $(1, -1)$. 4

(d) Find the area of the surface formed by the revolution of $y^2 = 4ax$ about the x-axis, by an arc from the vertex to one end of the latus rectum. 6

7. *(a)* *(i)* Find the reduction formula for $\int sin^n x\, dx$.

(ii) Evaluate $\int sin^4 x\, cos^2 x\, dx$.

3+2=5

(b) Find the length of the arc of the parabola $y^2 = 16x$ measured from the vertex to an extremity of the latus rectum. 3

(c) Show that the series $\sum \frac{(n+1)^n x^n}{n^{n+1}}$ is convergent if $x < 1$ and divergent if $x \geq 1$. 6

(d) Find the n^{th} derivative of $x^{n-1} log\, x$. 6
