53 (MA 401) NMCP

2017

NUMERICAL METHODS AND COMPUTER PROGRAMMING

Paper : MA 401

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Find a real root of $x^4 - x = 10$, correct to five decimal places by using Newton–Raphson method. 5

   (b) Evaluate : *(i)* $\Delta^2 x^3$ *(ii)* $\frac{\Delta^2}{E} x^3$, where E is shift operator and Δ is forward difference operator. 2+3=5

   (c) Write an algorithm to implement Euler's method. Using Euler's method, find a solution of the equation $\frac{dy}{dx} = y + x^2$, with initial condition $y = 1$ at $x = 0$ for the range $0 \le x \le 1$ in steps of 0·2. Compare the result with analytical solution. 4+3+3=10

2. *(a)* Using Bisection method, find a real root of $cos(x) = xe^x$ to correct upto three decimal places. 5

(b) Given, $sin\,45^\circ = 0{\cdot}7071$

$sin\,50^\circ = 0{\cdot}7660$

$sin\,55^\circ = 0{\cdot}8192$

$sin\,60^\circ = 0{\cdot}8660$

Find $sin\,52^\circ$ 7

(c) Using Modified Euler's method, find y at $x = 0{\cdot}1$ and $x = 0{\cdot}2$ given that $\frac{dy}{dx} = y - \frac{2x}{y}$, $y(0) = 1$ with correct result upto four places of decimals. 8

3. *(a)* Using Regula Falsi method, find a real root of $x^4 - 32 = 0$ to correct upto three decimal places. 5

(b) Compute $f'(0{\cdot}1)$ from the following data: 7

x :	0	1	2	3	4
$f(x)$:	1	0	1	10	13

(c) Using Runge–Kutta method of fourth order, solve $\frac{dy}{dx} = x^3 + \frac{y}{2}$ with $y(1) = 2$ at $x = 1{\cdot}1, 1{\cdot}2$. 4+4=8

4. (a) Using Gauss–elimination method solve

$$\begin{aligned} x + y + z &= 9 \\ 2x - 3y + 4z &= 13 \\ 3x + 4y + 5z &= 40 \end{aligned}$$

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(b) Given the following table :

x	:	0	1	2	5
$f(x)$	:	2	3	12	147

what is the form of the function ? 4

(c) Solve $\frac{dy}{dx} = 2e^x - y$ with $y(0) = 2$ at $x = 0{\cdot}1, 0{\cdot}2, 0{\cdot}3$ by Improved Euler Method and hence find $y(0{\cdot}4)$ and $y(0{\cdot}5)$ by Milne's method. 6+6=12

5. *(a)* Using Simpson's rule with four divisions $(n=4)$. Calculate the approximate value of $log 2 = \int_1^2 \frac{dx}{x}$. 6

(b) Solve the equations (by using Gauss–Seidel method) 7

$$27x+6y-z = 85$$
$$x+y+54z = 110$$
$$6x+15y+2z = 72$$

(c) Use Trapezoidal Rule with $n=4$ to estimate $\int_1^2 x^2 dx$. 7

6. *(a)* Define Principle of least square. Fit a second degree parabola to the following data :

x	:	1·0	1·5	2·0	2·5	3·0	3·5	4·0
y	:	1·1	1·3	1·6	2·0	2·7	3·4	4·1

2+10=12

(b) Write the Geometrical Interpretation of Newton-Raphson method. 4

(c) Using the iteration method solve $2x - log\, x = 7$ to correct upto six decimal places. 4
