Total number of printed pages–8

53 (MA 302) DSMA

2017

DISCRETE MATHEMATICS

Paper : MA-302

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

I. *(a)* If A, B and C are any three sets, then prove that

$$A-(B\cup C)=(A-B)\cap(A-C)$$ 4

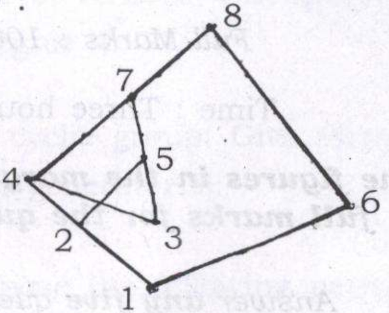
(b) If H and K are two normal subgroups of a group G, then prove that $H\cap K$ is also a normal subgroup of G. 5

Contd.

(c) Examine the satisfiability of the following compound proposition by using truth table 3

$$(p \to q) \wedge (p \to \neg q) \wedge (\neg p \to q) \wedge (\neg p \to \neg q)$$

(d) The Hasse diagram of the poset $(A, \leq)$, where $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, is given below :



Find the upper bounds of the subset $B = \{2, 3\}$. Does B possess lub ? Justify your answer. 2+2=4

(e) Test whether the following degree vector v is graphical or not. 4

$$v = [5\ 4\ 4\ 3\ 3\ 3\ 2]$$

2. (a) Using rule of inferences, show that $\neg p$ is a valid conclusion of the following premises : 5

$$\neg(p \wedge \neg q), \neg q \vee r, \neg r$$

(b) If $f:A \to B$ and $g:B \to C$ are two one-one onto functions, then prove that $g.f:A \to C$ is also one-one onto. 4

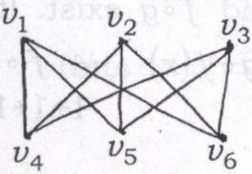
(c) Define group. Show that the set $G=\{1,\omega,\omega^2\}$, where ω is an imaginary cube root of unity, is a group with respect to multiplication. 2+4=6

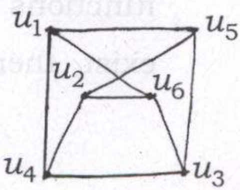
(d) Draw the Hasse diagram of the poset $(P(S),\leq)$, where $S=\{a,b,c\}$ and '$\leq$' is a partial order relation on $P(S)$ defined as $A \leq B$ iff $A \subseteq B$. Hence show that $(P(S),\vee,\wedge)$ is a lattice. 2+3=5

3\. (a) Simplify 5

$f(x_1,x_2,x_3,x_4)=x_1' x_2' x_3' x_4 + x_1' x_2 x_3' x_4 + x_1x_2x_3'x_4 + x_1x_2'x_3'x_4 + x_1x_2x_3'x_4' + x_1x_2x_3x_4 + x_1x_2x_3x_4'$ using K-map.

(b) Examine whether the following two graphs are isomorphic or not. 3





(c) Show that $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent. 3

(d) Define equivalence relation. If R is a relation defined on $\mathbb{N} \times \mathbb{N}$ by $_{(x,y)}R_{(z,t)} \Leftrightarrow x+t=y+z$, where $\mathbb{N}$ is the set of all natural numbers, then show that R is an equivalence relation on $\mathbb{N} \times \mathbb{N}$. 2+4=6

(e) Define alternating group. Give an example. 2

(f) Write the negation of $(\exists x)(Px)$. 1

4. (a) Show that the following compound proposition is a tautology :

$$((p \to r) \vee (q \to r)) \leftrightarrow ((p \wedge q) \to r)$$ 3

(b) If $f(x) = -|x|$ and $g(x) = \log x$, then determine whether the composite functions $g \circ f$ and $f \circ g$ exist. If they exist, then find $g \circ f(x)$ and $f \circ g(x)$. 1+1+1+1=4

(c) Show that if every element of a group is its own inverse, then G is abelian. 3

(d) Draw the simple undirected planar graph represented by the following adjacency matrix. 2

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

(e) Express $f(x,y,z)=(x+y)(x+y')(x'+z)$ in DNF and CNF. 2½+2½=5

(f) Can a tree exist with the following degree vector, $v=[1\ 4\ 2\ 2\ 4\ 3]$? Justify your answer. 2

(g) Show that $\neg p \wedge p$ is a contradiction. 1

5. (a) Show that a non-empty subset H of a group G is a subgroup of G if and only if

(i) $a \in H, b \in H \Rightarrow ab \in H$

(ii) $a \in H \Rightarrow a^{-1} \in H$

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(b) Translate the following sentences into logical symbols.

(i) Not all complex numbers are real numbers.

(ii) There are some seniors who like physics but not mathematics.

(iii) The trains run late on exactly those days when I take it.

(iv) All students who like mathematics are intelligent.

1+1+1+1=4

(c) Show that the sum of degrees of all vertices in a graph G is even. 4

(d) Find all the ordered pairs in a relation R on the set $A=\{1,2,3,4,5,6\}$, where R is defined as $R=\{(a,b)|a \text{ divides } b\}$. 1

(e) Draw a graph of each of the following :

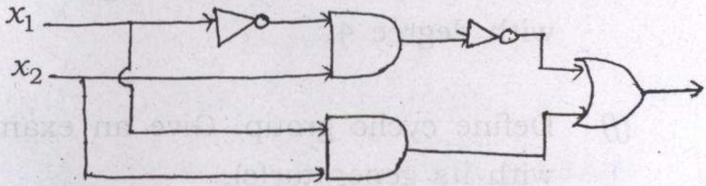
(i) Eulerian but not Hamiltonian

(ii) Hamiltonian but not Eulerian.

1½+1½=3

(f) Examine whether the set $\mathbb{Z}$ of all integers with respect to the relation '$\leq$' defined as $x \leq y$ iff $5 | x - y$ is a poset or not. 2

6. (a) Write the Boolean function of the following logic gate : 3



(b) State the contrapositive and inverse of the following propositions : 2+2=4

(i) If it snows today, I will not ski tomorrow.

(ii) I come to class whenever there is going to be a quiz.

(c) If the truth value of '$p \to q$' is F, determine the truth value of the following compound proposition :

$$(\neg p \vee q) \to (p \leftrightarrow \neg q)$$ 2

(d) Draw a simple cubic graph G with six vertices. Also find $\overline{G}$ (complement of G). 2+1=3

(e) Let G be a simple graph with 13 vertices and 21 edges. If G consists of vertices of degrees 3 and 4 only, find the number of vertices with degree 3 and with degree 4. 3

(f) Define cyclic group. Give an example with its generator(s). 3

(g) Decompose the following permutation into transpositions : 2

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 5 & 2 & 4 & 3 & 1 & 7 \end{pmatrix}$$
