Total number of printed pages–8

53 (EC 302) LSSG

2017

LINEAR SYSTEMS AND SIGNALS

Paper : EC 302

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions out of* ***seven.***

1. *(a)* Represent the following signal in terms of unit-step function. 3

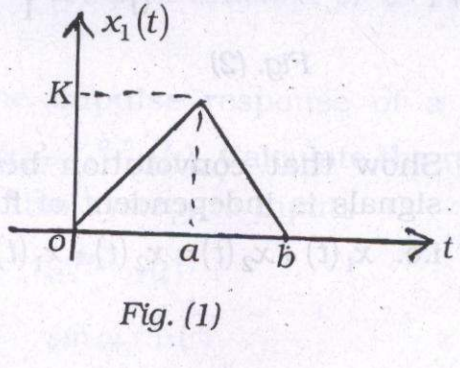

Fig. (1)

(b) Define impulse function. Show how an impulse function be approximated in terms of a rectangular pulse function. Describe the arbitrary signal $x(t)$ in terms of a series of impulse functions. 2+2+4=8

(c) The variation of two signals $x_1(t)$ and $x_2(t)$ are given in *Fig. (2)*. Evaluate $x_1(t) * x_2(t)$. 6

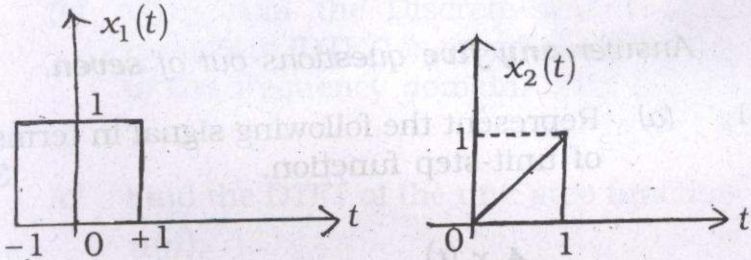

Fig. (2)

(d) Show that convolution between two signals is independent of its order i.e. $x_1(t) * x_2(t) = x_2(t) * x_1(t)$. 3

2. *(a)* Find $x(-2t+1)$ for the signal $x(t)$ shown in *Fig. (3)*. 4

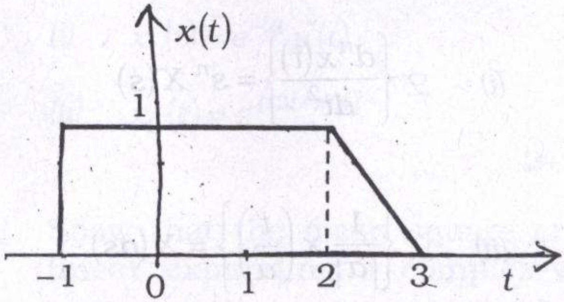

Fig. (3)

(b) Check whether the following systems are linear and time-invariant.

(i) $y(t) = t\sin(x(t))$

(ii) $y(t) = Re\{x(t)\}$ where, $x(t)$ is a complex function of t.

2+2=4

(c) The impulse response of a system $h(t) = e^{-\alpha_1 t}u(t)$. Calculate the response of this system to inputs

(i) $e^{-\alpha_2 t}u(t)$

(ii) $\sin\omega_0 t\, u(t)$

(iii) $t\,u(t)$ 3×4=12

3. *(a)* Prove the following if $X(s)$ is the bilateral Laplace transform of $x(t)$

(i) $\mathcal{L}\left\{\frac{d^n x(t)}{dt^2}\right\} = s^n X(s)$

(ii) $\mathcal{L}\left\{\frac{1}{|a|} x\left(\frac{t}{a}\right)\right\} = X(as)$

2×4=8

(b) Determine the zero-input response and zero-state response of an LTI system represented by the following differential equation.

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 6x(t)$$

Given $y(0^-) = 1$ and $\left.\frac{dy}{dt}\right|_{t=0^-} = 2.$

8

(c) State and prove final value theorem.

4

4. *(a)* Explain how to calculate energy and power of a signal. Determine the energy and power of the signals,

(i) $x_1(t) = e^{-2t} u(t)$

(ii) $x_2(t) = e^{j(2t+\pi/4)}$

2+2+2=6

(b) Show that the mean square error in linear expanding a complex-valued signal $f(t)$ in terms of an orthogonal set, $\{g_1(t), g_2(t), ...g_N(t)\}$ is minimized when the co-efficient of expansion,

$$C_n = \frac{\langle f(t), g_n(t)\rangle}{\langle g_n(t), g_n(t)\rangle}.$$ 6

(c) Find the exponential Fourier series and sketch the corresponding line spectrum of a half-wave rectified sine wave shown in *Fig. (4)*. 8

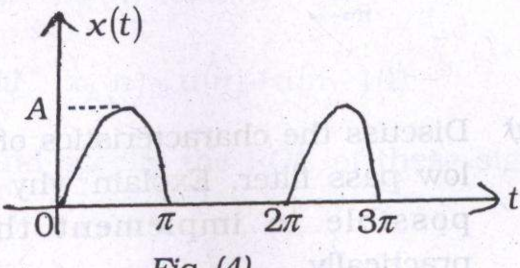

Fig. (4)

5\. (a) State and prove Parseval's theorem for power signals. 5

(b) Consider the rectangular pulse signal

$$x(t) = A\,rect\left(\frac{t}{2T_0}\right) = \begin{cases} A, & |t| < T_0 \\ 0, & |t| > T_0 \end{cases}$$

Find the Fourier transform of $x(t)$ and plot its magnitude and phase spectrum. 5

(c) Evaluate the Fourier transform of signum function. Using this result find the Fourier transform of $u(t)$, the unit step function. 5

(d) Find and sketch the Fourier transform of the impulse train,

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0).$$ 5

6\. (a) Discuss the characteristics of an ideal low pass filter. Explain why it is not possible to implement the same practically. 5

(b) Derive the relationship between the real and imaginary parts of the frequency response of a causal system. 5

(c) Show that $x(n) = z^n$ is an eigensignal of a Linear time-invariant discrete-time system. Evaluate the output, $y(n)$ for an LTI system if its impulse response, $h(n) = \left\{ \underset{\uparrow}{1}, \frac{1}{2}, \frac{1}{4} \right\}$ and input,

$x(n) = \left\{ \underset{\uparrow}{2}, 4, 8 \right\}$. 5

(d) Determine the z-transform of

(i) $x_1(n) = n\,u(n)$

(ii) $x_2(n) = u(n) - u(n-10)$

and specify the ROC of these signals. 5

7. *(a)* An LTI system is characterized by the *z*-transform of the impulse response, $h(n)$ as $H(z)=\frac{3-4z^{-1}}{1-3\cdot5z^{-1}+1\cdot5z^{-2}}$.

Specify the ROC and determine $h(n)$ for the following conditions :

(i) The system is causal and unstable

(ii) The system is non-causal and stable

(iii) The system is anti-causal and unstable. 8

(b) Show that the Discrete-time Fourier transform (DTFT) is a periodic function in the frequency domain. 4

(c) Find the DTFT of the unit step function $u(n)$. 8