53 (MA 101) ENMA-I

2017

ENGINEERING MATHEMATICS-I

Paper : MA 101

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Find the radius of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at (x,y). 7

(b) Find the volume of the solid formed by revolving the cycloid $x=a(\theta-\sin\theta)$ and $y=a(1-\cos\theta)$ about its base. 7

(c) If $\cos^{-1}\left(\frac{y}{b}\right)=\log\left(\frac{x}{n}\right)^n$, show that

$$x^2y_{n+2}+(2n+1)xy_{n+1}+2n^2y_n=0.$$

6

2. *(a)* Show that the series $\sum_{n=1}^{\infty} \frac{(n+1)^n . x^n}{n^{n+1}}$ is convergent if $x < 1$ and divergent if $x \geq 1$. 6

(b) Prove that the lines $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z-4}{5}$ and $2x - 3y + z = 0 = x + y + 2z + 4$ are coplanar. Find also their point of intersection. 5+1=6

(c) If (l_1, m_1, n_1) and (l_2, m_2, n_2) be the direction cosines of two perpendicular lines. Then prove that the direction cosines of the line perpendicular to both of them are $m_1 n_2 - m_2 n_1$, $n_1 l_2 - n_2 l_1$ and $l_1 m_2 - l_2 m_1$. 4

(d) Using comparison test, show that the series $\sum_{n=1}^{\infty} \frac{n+1}{n.3^{n+1}}$ is convergent. 4

3. *(a)* Solve : ***(any three)*** 3×4=12

(i) $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 18y = 0$,

$y(0) = 0$, $y'(0) = 0$, $y''(0) = 2$

(ii) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = e^x \cosh 2x$

(iii) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x$

(iv) $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$

(b) Form the differential equation from : 4+4=8

(i) $Ax^2 + By^2 = 1$

(ii) $Ae^{3x} + Be^x = y$

4. (a) Solve : 4×2=8

(i) $y = 2px + yp^2$

(ii) $y = (x-a)p - p^2$

(b) Is the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\lfloor n}$ converge absolutely ? Justify. 5

(c) Find the equation of the evolute of the parabola $y^2 = 4ax$. 7

5. *(a)* Find the asymptotes of the curve

$$x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$$ 6

(b) Show that the length of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ is $6a$. 6

(c) Solve : ***(any two)*** 4+4=8

(i) $(x+1)\dfrac{dy}{dx} = x\left(y^2+1\right)$

(ii) $\left(x^2 - y^2\right)dx - xy\,dy = 0$

(iii) $e^x \tan y\,dx + \left(1 - e^x\right)\sec^2 y\,dy = 0$

6. *(a)* Find the equation of the plane passing through the middle point of the join of the point (2,–3,1) and (4,5,–3) and in perpendicular to the line joining the points. 5

(b) Find the equation of the sphere through the points (0,0,0), (1,–1,0), (2,0,–2) and (0,1,2). 5

(c) Solve the simultaneous equations

$$\frac{dy}{dt} = 3x - 2y\,; \quad \frac{dx}{dt} = -2x + 3y$$ 4

(d) Show that the function $f(x,y) = 2x^4 - 3x^2y + y^2$ does not have a maximum or a minimum at (1,1). 6
