53 (MA 401) NMCP

2016

NUMERICAL METHODS AND COMPUTER PROGRAMMING

Paper : MA 401

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* Write the fundamental theorem of roots. Using Bisection method solve, $x^3 - 4x - 9 = 0$ correct upto three decimal places. 1+5=6

   (b) Write an algorithm to implement Euler's method. Solve the differential equation $\frac{dy}{dx} = x^2 + y^2$ with $y(0) = 1$, using Euler's method by choosing a step size of $h = 0{\cdot}2$ to obtain solution for $0 \le x \le 1$. 4+8=12

(c) Find the third divided difference with arguments 2, 4, 9, 10 of the function $f(x) = x^3 - 2x$. 2

2. (a) Using iteration method solve $e^x - 4x - 1 = 0$ to correct upto five decimal places. 5

(b) State Newton's Backward interpolation formula. Using it find the value of $f(7{\cdot}5)$ from the following data :

x	1	2	3	4	5	6	7	8
$f(x)$	1	8	27	64	125	216	343	512

2+5=7

(c) Using Improved Euler's method, find y at $x = 0{\cdot}1$ and $x = 0{\cdot}2$ given that $\frac{dy}{dx} = y - \frac{2x}{y}$, $y(0) = 1$. 8

3. (a) Using Runge-Kutta method of fourth order, solve, $\frac{dy}{dx} = xy + y^2$ with $y(0) = 1$ at $x = 0{\cdot}1, 0{\cdot}2, 0{\cdot}3$. 12

(b) State Lagrange's interpolation formula. Using it find the form of the function from the following data: 2+6=8

x :	3	2	1	−1
$f(x)$:	3	12	15	−21

4. (a) Using Newton-Raphson method, find a positive root of $x^4 - x = 10$ to correct upto fourth decimal places. 5

(b) Given $log_{10}(654) = 2{\cdot}8156$,

$log_{10}(658) = 2{\cdot}8182$,

$log_{10}(659) = 2{\cdot}8189$,

$log_{10}(661) = 2{\cdot}8202$, find the value of $log_{10}(656)$. 5

(c) Write the principle of least squares, Using this method fit the equation $y = a + bx + cx^2$ to the following data:

x :	1	2	3	4	5	6	7
y :	2·31	2·01	3·80	1·66	1·55	1·47	1·41

1+9=10

5. (a) Using Gauss-elimination method solve:

$$x + 4y - z = 5$$

$$x + y - 6z = -12$$

$$3x - y - z = 4$$

5

(b) Find the first and second derivatives of the function tabulated below at the point $x = 1{\cdot}5$ 4+3=7

x	:	1·5	2·0	2·5	3·0	3·5	4·0
$f(x)$	:	3·375	7·000	13·625	24·000	38·875	59·000

(c) Solve $\frac{dy}{dx} = \frac{1}{2}(1+x)y^2$ with $y(0) = 1$ at $x = 0{\cdot}2, 0{\cdot}2, 0{\cdot}6$ by an appropriate method and hence find $y(0{\cdot}8)$ by Milne's method. 8

6. (a) Using Regula Falsi method, find a real root of $\cos x = xe^x$ to correct upto fourth decimal places. 7

(b) Using Gauss-Seidel method solve : 8

$$2x + y + 6z = 9$$
$$8x + 3y + 2z = 13$$
$$x + 5y + z = 7$$

to correct upto three decimal places.

(c) Evaluate $\int_{0{\cdot}5}^{0{\cdot}7} x^{\frac{1}{2}} e^{-x} dx$ approximately by using Simpson's $\frac{1}{3}$rd rule. 5