Total number of printed pages–7

53 (MA 302) DSMA

2016

DISCRETE MATHEMATICS

Paper : MA 302

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* If A, B, C are any three sets, then prove that 3+2=5

(i) $A\times(B\cap C)=(A\times B)\cap(A\times C)$

(ii) $(A\cup B)^C = A^C \cap B^C$

Contd.

(b) Define Group. Show that the set $G = \{a + b\sqrt{2} : a, b \in Q\}$, where Q is the Set of rational numbers, is a group with respect to addition. 2+4=6

(c) Prove that the mapping $f : A \to B$ is one-one onto if and only if there exists a mapping $g : B \to A$ such that gof and fog are the identity mapping on A and B respectively. 4

(d) Define a subgroup of a group. If H and K are subgroups of a group G, then show that HK is a subgroup of G if and only if $HK = KH$. 5

2. (a) Show that for any graph with six points G or $\overline{G}$ contains a triangle. 7

(b) Define adjacency matrix for undirected graph. Draw the undirected graph represented by adjacency matrix A given by 2+3=5

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

(c) Obtain the principal disjunctive normal form of 4

$$(p \wedge \sim q \wedge \sim r) \vee (q \wedge r)$$

(d) Translate the following sentences into logical expressions : 1×4=4

(i) Every person is precious.

(ii) Any integer is either positive or negative.

(iii) Some students are not successful.

(iv) All students in a class have not taken discrete mathematics.

3. *(a)* Show that every cubic graph has an even number of points. 5

(b) Show that the set of all positive integers under divisibility relation form a poset. 5

(c) Consider the set $I\!N \times I\!N$, the set of ordered pairs of natural numbers. Let R be a relation on $I\!N \times I\!N$ which is defined by $(a, b)\ R(c, d)$ if and only if $ad = bc$. Prove that R is an equivalence relation on $I\!N \times I\!N$. 5

(d) Show that if a and b are any two elements of a group G, then $(ab)^2 = a^2b^2$ if and only if G is abelian. 5

4. *(a)* Define normal subgroup. Show that a subgroup H of a group G is normal if and only if $xHx^{-1} = H \ \ \forall x \in G$. 1+4=5

(b) Write down all the permutations on four symbols 1,2,3,4. Which of these permutations are even ? 3+2=5

(c) Consider the set $S = \{a,b,c\}$. Prove that $(P(S),\subseteq)$ is a poset. Draw the Hasse diagram of the poset $(P(S),\subseteq)$. 4+2=6

(d) Show that a lattice is a partially ordered set. 4

5. (a) Is the relation "is brother of" an equivalence relation on a set of all human beings ? Why ? 3

(b) If G is a group, then prove that for every $a,b \in G$, $(ab)^{-1} = b^{-1}a^{-1}$. 3

(c) Let R_0, denote the set of all non-zero real numbers. Prove that the map $f: R_0 \to R_0$ defined by $f(x) = \frac{1}{x}, \forall x \in R_0$ is bijective. 4

(d) Examine whether the following compound proposition is tautology *or* not. 2

$$(p \to q) \leftrightarrow (\sim q \to \sim p)$$

(e) Find the negation of each of the following quantified predicates 2+2=4

(i) $(\exists x D)(x+2=7)$

(ii) $(\forall x \in D)(x+3<10)$, where $D=\{1,2,3,4\}$

(f) Show that in a non-directed graph, the total number of odd degree vertices is even. 4

6. (a) Establish the equivalence 3+2=5

$$p \rightarrow q \vee r \equiv p \wedge (\sim p) \rightarrow r \equiv p \wedge (\sim r) \rightarrow q$$

Hence write the following sentence in *two* different ways. If n is prime, then n is odd *or* n is 2.

(b) If L be a lattice then for every $a, b \in L$ prove that 5

(i) $a \vee b = b$ iff $a \leq b$

(ii) $a \wedge b = a$ iff $a \leq b$

with usual meaning of the symbols.

(c) Let A and B be two non-empty sets. If $f : A \to B$ is invertible, then show that f^{-1} is also invertible. 3

(d) Find the order of each element of the multiplicative group $\{1, -1, i, -i\}$. 2

(e) Prove that every cyclic group is abelian. But the converse may not be true in general. Give an example to show that every abelian group is not cyclic. 5
