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53 (MA 301) ENMA-III

2016

ENGINEERING MATHEMATICS-III

Paper : MA 301

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Show that the function

$$u = x^2 - y^2 - 2xy - 2x - y - 1$$

is harmonic. Find the conjugate harmonic function v and express $u + iv$ as a function of z where $z = x + iy$. 5

(b) Solve : ***(any one)*** 5

(i) $q^2 = z^2 p^2 (1 - p^2)$

(ii) $p^2 + q^2 = x^2 + y^2$

Contd.

(c) If $L\{F(t)\} = f(s)$ then prove that

$$L\{e^{at}F(t)\} = f(s-a),\ s > a$$ 5

(d) (i) If Sik is symmetric and Aik is skew-symmetric then prove that $Sik\,Aik = 0$.

(ii) If a contravariant tensor of rank two is skew-symmetric in one co-ordinate system then it is so in every coordinate system.

3+2=5

2. (a) Prove that $L\{t^n\} = \frac{n!}{S^{n+1}},\ s > 0,\ n = 0,1,2....$

5

(b) Using Charpit's method solve

$$(p^2 + q^2)y = qz$$ 6

(c) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region

(i) $1 < |z| < 2$

(ii) $|z| > 2$ 2+2=4

(d) Find the fundamental conjugate tensor for the line element given by

$$ds^2 = dx^2 + dy^2 + dz^2 - 2dxdy + dydz - dzdx$$

5

3. (a) Evaluate the integral

$$\int_0^{1+i} \left(x - y - ix^2\right) dz$$

along the path

(i) $y = x$

(ii) along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to imaginary axis from $z = 1$ to $z = 1 + i$. 2+4=6

(b) Prove that $L\left\{\frac{\cos at - \cos bt}{t}\right\} = \frac{1}{2} \log \frac{s^2 + b^2}{s^2 + a^2}$

5

(c) Form the partial differential equations of 3×3=9

(i) $z = ax + by + a^2 + b^2$

(ii) $z = yf(x) + xg(y)$

(iii) $f\left(x^3 - y^3, x^2 - z^2\right) = 0$

4. *(a)* *(i)* Assume $\phi = a_{jk} A^j A^k$. Then show that $\phi = b_{jk} A^j A^k$ where b_{jk} is symmetric.

(ii) If $C(m,n)$ is the cofactor of A_{mn} in $\det(A_{mn}) = d \neq 0$ and $A^{mn} = \frac{C(m,n)}{d}$ then show that $A_{mn} A^{in} = \delta^i_m$.

(iii) If $A_{ij} = 0$ for $i \neq j$ and $A_{ij} \neq 0$ for $i = j$ then show that the conjugate or reciprocal tensor $B^{ij} = 0$ for $i \neq j$ and $B^{ii} = \frac{1}{A^{ii}}$ (no implied summation) 3+3+3=9

(b) Solve : ***(any one)*** 5

(i) $$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^2 z}{\partial x^2 \partial y} = 3x^2 y$$

(ii) $$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = \cos(2x + y)$$

(c) Find Laplace transform of the following functions : ***(any two)*** 3×2=6

(i) $4t^3 - e^{-2t}$

(ii) $e^{-3t}\left(t^3 + 2\right)^3$

(iii) $t^2 \sin t$

5. *(a)* *(i)* If $A_k^{ij} B_i C_j D^k$ is an invariant for arbitrary covariant vectors B_i, C_j and contravariant vector D^k, show that A_k^{ij} is a mixed tensor of rank 3. 3

(ii) Prove that the Kronecker Delta is a mixed tensor of rank 2. 2

(b) Solve : ***(any two)*** 2×3=6

(i) $\left(x^2 - y^2 - z^2\right)p + 2xyq = 2xz$

(ii) $x\left(y^2 - z^2\right)p + y\left(z^2 - x^2\right)q = z\left(x^2 - y^2\right)$

(iii) $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$

(c) Show that the function $f(z) = z|z|$ is not analytic anywhere. 4

(d) Determine the poles of $f(z)$ and residues at its poles

$$f(z) = \frac{z-1}{(z+1)^2(z-2)}$$

and hence evaluate $\int_C f(z)dz$ where C is the circle $|z-i| = 2$. 5

6. (a) Evaluate : ***(any two)*** 4+4=8

(i) $L^{-1}\left\{\frac{3}{3s-5} - \frac{4+3s}{3s^2-12} + \frac{2-4s}{9s^2+16}\right\}$

(ii) $L^{-1}\left\{\frac{4s+12}{s^2+8s+16}\right\}$

(iii) $L^{-1}\left\{\frac{se^{-\frac{4\pi s}{3}}}{s^2+25}\right\}$

(b) (i) If $a_{ijk}\ dx^i dx^j dx^k = 0$ for all values of a_{ijk} then show that

$$a_{123} + a_{132} + a_{213} + a_{231} + a_{312} + a_{321} = 0$$

4

(ii) If A_{ij} is skew-symmetric then show that 2

$$\left(B_j^i B_n^m + B_n^i B_j^m\right) A_{im} = 0$$

(c) Evaluate the complex integral

$$\int_C \frac{z}{\left(z^2 - 3z + 2\right)} dz$$

where C is the circle $|z - 2| = \frac{1}{2}$ 6

7. (a) Solve using Laplace transform

$$y'' - 3y' + 2y = 4e^{2t}$$

with $y(0) = 3$ and $y'(0) = 5$ 5

(b) (i) State Einstein's Summation Convention. Differentiate between a dummy index and a real index.

2

(ii) Prove that $A^j \delta_j^i = A^i$ 1

(c) Find the singularities of the following functions and state their nature and location. 2+2=4

(i) $\dfrac{z - \sin z}{z^2}$

(ii) $\dfrac{1}{\cos z - \sin z}$

(d) Find the image of $|z - 3i| = 3$ under the mapping $w = \frac{1}{z}$. 3

(e) Using the method of separation of variables, solve $\dfrac{\partial u}{\partial x} = 2\dfrac{\partial u}{\partial t} + u$ where $u(x,0) = 6e^{-3x}$. 5

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