Total number of printed pages–6

53 (EC-302) LSLG

2016

LINEAR SYSTEMS AND SIGNALS

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions out of **seven.***

1. *(a)* Describe the condition under which a discrete-time signal is periodic. Give *any two* examples of periodic exponential signals. Which are discrete in time? 4

   (b) Determine whether the systems represented by the following input-output relationships are linear time-invariant (LTI) systems.

   (i) $y(n) = nx(n)$

   (ii) $y(t) = x(2t)$ 6

Contd.

(c) Show that the mean square error in approximating a given function, $f(t)$ in terms of an orthogonal set, $\{g_1(t), g_2(t) \cdots g_N(t)\}$ is minimized when

$$f(t) \approx \sum_{i=1}^{N} c_i g_i(t),$$

where $C_i = \dfrac{< f(t), g_i(t) >}{< g_i(t), g_i(t) >}$ 10

2. (a) Define impulse signal, $\delta(t)$ in the continuous time domain. Show that the convolution of a signal with $\delta(t - t_0)$ gives us a shifted signal. 5

(b) Show that the zero-state response of an LTI system can be obtained as $y_{zs}(t) = x(t) * h(t)$, where $x(t)$ is the input and $h(t)$ is the impulse response of the system.

{Hint : start with the pulse wave approximation of $x(t)$ }. 7

(c) Find the convolution sum between the signals, $x_1(t) = \left\{ \underset{\uparrow}{1}, 2, 3, 4 \right\}$ and

$$x_2(t) = \left\{ 1, 1, \underset{\uparrow}{0}, -1, -1 \right\}$$ 4

(d) Show that the even harmonics are absent in the Fourier series expansion of a periodic signal with half-wave symmetry. 4

3. (a) Find the exponential Fourier series and plot the line spectrum of a half-wave rectified sine wave whose one period is written as,

$$x(t) = \begin{cases} A \sin t\,; & 0 \le t \le \pi \\ 0\,; & \pi < t < 2\pi \end{cases}$$ 8

(b) Show that by passing this signal through an L-section (inductor) filter, one can improve the ripple factor. 6

(c) Find the exponential Fourier series and sketch the corresponding spectra for the impulse train, $x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$.

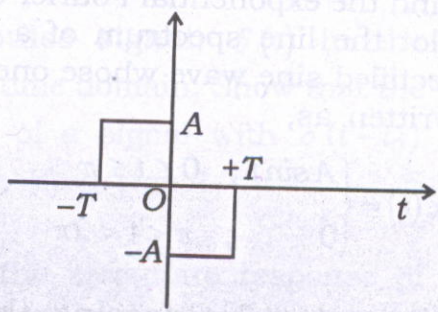
6

4. *(a)* Derive the Fourier transform of a unit-step function. 6

(b) Prove Parseval's identity for a discrete-time periodic signal. 6

(c) Show that power spectral density (PSD) and auto-correlation function are Fourier transform pairs. 8

5. *(a)* Find the magnitude and phase spectrum of the finite duration signal shown below : 10



(b) Show that for a causal system with frequency response $H(\omega)$, the magnitude $|H(\omega)|$ and phase $\angle H(\omega)$ are inter dependent. 10

6. *(a)* Find the Laplace transform of the signal $x(t) = t^n u(t)$. 6

(b) Show that an LTI system is stable if the impulse response $h(t)$ is absolutely integrable

i.e. $\int_{-\infty}^{\infty} |h(t)| \cdot dt < \infty$ 4

(c) Using unilateral Laplace transform, determine the output of a system represented by the differential equation

$$\left(D^2 + 5D + 6\right) y(t) = (D + 6)\, x(t)$$

where $D = \frac{d}{dt}$. The input $x(t) = u(t)$ and the initial conditions $y\left(0^-\right) = 1$ and $\dot{y}\left(0^-\right) = 2$. Identify the zero state response $y_{zs}(t)$ and zero-input response $y_{zi}(t)$ of the system. 10

7. (a) Verify the final value of $x(t) = \left(2 + e^{-3t}\right) u\,(t)$ using final value theorem. 4

(b) Derive the Z-transform of the signal $x(n) = a^n u(n)$. Using this result find out the inverse Z-transform of $\frac{1}{z-a}$. 6

(c) Find the inverse Z-transform of

$$X(z) = \frac{1}{1 - 1 \cdot 5z^{-1} + 0 \cdot 5z^{-2}}; \quad |z| > 1$$

using the contour integration method. 10
