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53 (MA 201) ENMA-II

2016

ENGINEERING MATHEMATICS-II

Paper : MA 201

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *qustions.*

1. *(a)* Calculate the median from the following : 3

Class interval :	0-10	10-20	20-30	30-40	40-50
Frequency :	5	6	8	30	10

(b) If $P(A)=a$ and $P(B)=b$, then show that $P\left(A/_B\right)\geq\frac{a+b-1}{b}$. 2

Contd.

(c) Find the rank of the matrix

$$\begin{pmatrix} 2 & 3 & -2 & 4 \\ 3 & -2 & 1 & 2 \\ 3 & 2 & 3 & 4 \\ -2 & 4 & 0 & 5 \end{pmatrix}$$

by row elementary transformation. 7

(d) Write the condition for a Fourier expansion. Find a Fourier series of the following function :

$$f(x) = \begin{cases} x^2, 0 < x \leq \pi \\ -x^2, -\pi < x \leq 0 \end{cases}$$ 3+5=8

2. (a) Urn I has 2 white and 3 black balls, urn II has 4 white and 1 black balls and urn III has 3 white and 4 black balls. An urn is selected at random and a ball drawn at random is found to be white. Find the probability that urn I was selected. 5

(b) Prove that

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{Cosnx}{n^2}, \; -\pi < x < \pi.$$

4

(c) Reduce the matrix 6

$$A = \begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

to its normal form.

(d) Find $\vec{r}$, if $\frac{d^2\vec{r}}{dt^2} = \vec{a}t + \vec{b}$, where $\vec{a}$ and $\vec{b}$ are constant vectors and given that both $\vec{r}$ and $\frac{d\vec{r}}{dt}$ vanish when $t=0$. 5

3. (a) Find the inverse of the following matrix by using Cayley-Hamilton theorem. 5

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

(b) Find the total work done in moving a particle in the force field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + 2\hat{k}$ along the straight line joining the points (0,0,0) and (2,1,3). 5

(c) A continuous random variable X has a probability density function $f(x) = kx^2e^{-x}$, $x \geq 0$. Find the value of k and the mean of X. 1+2=3

(d) If $u = x^3 - 2y^2$, $v = 2x^2 - y^2$ where $x = r\cos\theta$ and $y = r\sin\theta$, then show that $\frac{\partial(u,v)}{\partial(r,\theta)} = 6r^3 \sin 2\theta$. 3

(e) Prove that $\beta(m, \tfrac{1}{2}) = 2^{2m-1}\beta(m,m)$. 4

4. (a) If $\vec{a}$ is a constant vector, then prove that $div(\vec{r} \times \vec{a}) = 0$ and curl $(\vec{r} \times \vec{a}) = -2\vec{a}$. 5

(b) If $u = 3x + 2y - z$, $v = x - 2y + z$ and $w = 2(x + 2y - z)$, then show that they are functionally related and also find the relation. 4

(c) If X is a random variable then show that $\mu_2 = \mu'_2 - {\mu'_1}^2$, where μ_r is the rth central moment and μ'_r is the rth raw moment of X for $r = 0, 1, 2$. 4

(d) If A is idempotent matrix and $A+B=1$, then show that B is idempotent matrix and $AB = BA = 0$. 2+2=4

(e) A coin is tossed until a head occurs. Find the mathematical expectation of the number of tosses required. 3

5. (a) Find a Fourier cosine series of the function $f(x) = \pi - x$, where $0 < x < \pi$. 5

(b) If the mean and variance of the Binomial distribution are 6 and 1·5 respectively, then find $E[X - P(X \geq 3)]$, where X is a discrete random variable. 5

(c) If $\vec{A} = (2x^2y - x^4)\hat{i} + (e^{xy} - y\sin x)\hat{j} + x^2\cos y\,\hat{k}$, find $\frac{\partial \vec{A}}{\partial x}$. 5

(d) Find the characteristic roots and corresponding characteristic vectors for the matrix 2+3=5

$$\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

6. (a) If $\vec{r} = a\cos t\,\hat{i} + a\sin t\,\hat{j} + bt\hat{k}$, find

$$\left[\frac{d\vec{r}}{dt}\,\frac{d^2\vec{r}}{dt^2}\,\frac{d^3\vec{r}}{dt^3}\right]$$ 6

(b) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. 5

(c) Prove that any real 2×2 normal matrix is either symmetric or the sum of a scalar matrix and a skew—symmetric matrix. 5

(d) Determine the values of x, y and z when

$$\begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix}$$ 4

is orthogonal.
